



PHD

## The Payment Form Threshold in Mergers and Acquisitions - A Real Options Approach

Yin, Liang

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The payment form threshold in mergers and acquisitions  
– a real options approach

Liang Yin

A thesis submitted for the degree of Doctor of Philosophy  
University of Bath  
School of Management  
August 2008

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### Disclaimer

Please note that all the theoretical models in the thesis, i.e., Chapter 2, 3 and 4, are developed and written by myself, with the support of my supervisor Dr Andreas Krause in terms of the guidance and suggestion. The empirical investigation in Chapter 5 is a joint work by Dr Andreas Krause, Dr Ahmad Ismail (College of Business and Economics, United Arab Emirates University) and myself. Dr Ahmad Ismail provided me with access to his database of mergers and valuable suggestions and comments. I have conducted the empirical regression guided by Dr Andreas Krause. Dr Andreas Krause also contributed partially to the writing of the chapter (around 50%)

Liang Yin



## Abstract

In recent years, practitioners and academics have become increasingly concerned that traditional discounted cash flow valuation models, such as the net present value model, are not capable of adequately capturing the value of managerial flexibilities to delay, grow, scale down, or abandon projects. The effect of ignorance of such managerial flexibilities can be potentially substantial, with the possibility of producing biased decisions. Real options analysis provides the insights that business investment projects can be conceptually compared to financial options and is therefore able to seize the value of managerial flexibilities.

The purpose of this thesis is to develop a theoretical model based on option pricing theory to evaluate the managerial flexibilities arising in a variety of mergers and acquisitions, which vary in payment forms. The thesis shows how transactions can be structured as a real exchange options, given the share price of each participating firm is subject to a specified degree of uncertainty. The takeover decisions of bidder or target, i.e., the takeover threshold to bid or to accept the bid, is obtained through the analysis. In addition, the thesis provides valuable theoretical insights into the following aspects:

- The impact of the form of payment on the decision making process for each participant and corresponding merger terms
- The payment form that minimizes the threshold to trigger a transaction
- The allocating rule of mergers and acquisitions synergy when payment form threshold is employed

In the latter part of thesis, an empirical study is conducted on mergers and acquisitions completed by US public bidders between January 1985 and April 2004 excluding all financial institutions deals. Strong support is found from the data that some of the target firm characteristics such as expected growth rate and volatility are significant in explaining the payment form choices.

## Introduction

When an interested bidder approaches a target firm, the payment can be made in various forms. Two basic methods of payment include a pure shares offering or a pure cash offering. In a shares offering, the company shares of bidder firm are provided in exchange for shares in the target company. A cash offering is done in a more straightforward manner: the bidder simply pays a certain amount of cash, agreed upon before the deal, to gain the control of the target firm. In many cases, however, a blend of cash and shares is utilized and it is referred to as a mixed offering.

One of the implications from the Nobel winning Modigliani-Miller capital structure irrelevance principle is that in a perfectly efficient market - i.e., in the absence of taxes, bankruptcy costs and asymmetric information - payment form should not matter. The research exploring the rationale of different payment forms however relaxes one or a few assumptions that Modigliani and Miller lay down for the market. Asymmetric information, tax, agency problems, market friction alongside other theories have been proposed to explain the existence of payment form variation.

This thesis is also one of the attempts to answer that fundamental question: why does payment form matter in a takeover deal? In the mean time, it explores the economic impacts of various payment forms on the success or, in many cases, the failure of mergers and acquisitions transactions and how payment form choices affect the takeover decisions for all participants.

The model develops within a market friction framework. It is assumed that cash and shares are not freely convertible due to high transaction costs so that when a particular payment form is presented, the takeover participants are “stuck” with this payment form and its takeover decision must base on this particular payment form. What justifies this assumption is in reality the market impact when one tries to purchase/sell a large amount of shares during a very short period can be so large that it becomes unfeasible to do so.

Given that takeover participants stick to whatever payment form is presented, the next question is how payment forms affect their decisions. In short, the rationale is when presented with different payment forms, takeover participants face different level of “uncertainty” associated with the transaction payoff, leading to different optimization strategy driven by their respective takeover option. In other words, the thesis establishes a direct link between strategic considerations of takeover participants, captured by a real options analysis, and the payment form used in the deal. Given that a majority of previous research focuses on exploring the optimal timing of the takeover deal by taking into no account of the impact

from payment form, this direct link can be deemed as one of the main theoretical contributions of the thesis. It contributes to the research ground investigating rationale for the medium of exchange in corporate deals by being one of the very first attempts to introduce real options framework.

Although a significant part of the thesis is dedicated to investigate that direct link in a more formal fashion, it is my aim to provide here the basic rationale in laymen's term.

An option to wait suggests investor can make their investment at any time at the right price, i.e., flexibility has value. The thesis assumes following the literature that the bidding firm holds a real option to purchase the target firm and the target firm holds a real option to sell its own firm.

One of most widely discussed real options implications is that uncertainty affects the investment decision. What does uncertainty mean? It essentially suggests that there are various possible scenarios that can play out in the future therefore one can potentially benefit from waiting to make a better decision. Consider an investor who has the right to purchase an asset (which changes value over time randomly) worth of 101 pounds with 100 pounds cash. If the investor decides to complete this investment, he will receive immediately one pound of profit. However the investor is aware that this asset can potentially increase value in the future and might for example rise to 120 pounds that yields a potential 20 pounds of profit. A rational investor should therefore account for the probability of such scenarios when making the decision. This is how real options optimization process works. Real options analysis suggests there is a threshold beyond which the investor should optimally exercise his option - i.e., complete the investment - and take the immediate profit. Let me assume the threshold is 10 pounds in this case. As a result, if the investor finds himself in a situation where he can pay 100 pounds cash to purchase the asset mentioned above that is worth 110 pounds, he should stop waiting and complete the deal.

Interestingly, if the same investor wants to purchase another asset, of which value also changes over time randomly but governed by different source of randomness, he might find himself in the situation where he will need for example an extra 15 pounds of profit to give up the right to wait. The reason is that with this different asset, the possible scenarios that could happen vary and so do the probabilities associated with each of those scenarios. The real options analysis will suggest a different threshold for the investor to exercise the option.

To this point, it is clear that "uncertainty affects decisions" means when asset changes value over time, it creates uncertain payoff for the decision maker to purchase/sell this particular asset. Using a real options analysis, the decision

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maker therefore requires an extra amount of profit (i.e., the threshold) to give up the flexibility. And the different sources of randomness associated with the asset are likely to result in different real options thresholds.

The analysis for participant's decision in a corporate takeover deal is not that different. The complicating factor is that for most cases, there are two sources of randomness involved. For example, in a shares offering, target firm is essentially exchanging one random asset - his own firm's shares - with another random asset, the bidder firm's shares. While the math needed to solve the optimal strategy becomes more complex, the rationale stays the same - uncertainty drives decisions.

Let me present another illustrative example. Target firm's total shares are currently valued at 1,000,000 pounds and bidder firm can offer either cash or shares to purchase all the outstanding shares. According to the analysis above, the target firm understands that his option to sell has value and this value needs to be compensated. Let me assume that target requires 10% more to justify his option when he sells his firm against cash. The important point is that this value varies in accordance with the payment form presented. An extreme example should help illustrate the idea. Let me further assume that bidder firm's shares are identical to target firm's shares. When the bidder firm's shares are presented to exchange for target firm's shares, there is in fact no uncertainty associated with the potential payoff target firm can obtain from the deal. If 1,000,000 pounds of bidder firm's shares are offered to exchange for 1,000,000 pounds of target firm's shares, despite the fact that the value of these shares still fluctuates over time, it will stay as a profitless deal for target firm forever. In other words, the value of options to wait for the target firm is essentially zero as there is no potential benefit from waiting. In this case, target will need far less than 10% to enter the deal.

It should be clear now that different payment forms will result in different thresholds for target firm to justify his own real option. The same rationale applies to the bidder. To summarize, from a pure real options perspective, payment form matters for takeover decisions. The thesis presents this idea in a more mathematical manner.

Before proceeding to the main content, it is my aim to provide further information to justify the methodology that will be used throughout the whole thesis: real options analysis. Since the term is introduced, real options thinking has emerged as a significant influence on analysis in mergers and acquisitions. BRUNER (2004) highlights four reasons why analysts and executives today should strive to employ real options thinking in a merger deal:

- Real options are easy to understand. Real options is actually interchangeable with notions such as “rights”, “flexibility,” or “commitments” that are frequently discussed by corporate managers.
- Real options value can be specially considerable for a firm that is growing, with the ability to do things other firms cannot, and/or has unique assets. For some industries such as technology, pharmaceuticals, and aerospace, where there exists the pervasiveness of real options, the real options value can easily exceeds half of the total value of the firm and therefore cannot and should not be ignored.
- Executives and mergers and acquisitions deal designers can easily create and destroy real options value, with a large impact on careers.
- Real options thinking is able to capture effects that the discounted cash flow method cannot show. Apart from managerial flexibility, discounted cash flow analysis is also criticized for not being able to reflect qualities about an asset that are not considered in projected cash flows. As a result, discounted cash flow analysis alone often leads to inaccurate estimation of the asset value.

The remainder of the thesis is organized as follows.

Chapter 1 provides a review on the current literature in two main aspects: mergers and acquisitions and real options. Motives for mergers and acquisitions, empirical performance of deals, valuation methods, takeover bidding theories, and methods of payment are covered in the first subsection, followed by a review of recent contributions to real options theory. The chapter concludes with the research questions to be discussed throughout the thesis.

Chapter 2 presents a basic model, providing fundamental framework for all the models discussed subsequently.

Chapter 3 and 4 both focus on the payment form threshold. Two models vary in assumptions in terms of synergy created through the deal and other aspects of deal design and therefore require different solving process. However, the results demonstrate strong consistency.

Chapter 5 is the empirical investigation of the predictions generated.

Chapter 6 concludes the whole thesis. It summarizes main findings, empirical and theoretical contributions, implications of the results, research limitations, and future research directions. The rest part is bibliography and appendixes.

## Chapter 1

# Literature review and research questions

## 1.1 Mergers and acquisitions

### 1.1.1 Terminology

Given that the definitions for mergers and acquisitions activities are not unique and sometimes vary considerably between authors, it is necessary to provide a clear definition at the beginning of the thesis.

Following WESTON ET AL. (1998), I define *mergers* as the process of combining two or more companies, generally by offering the stockholders of one company with securities in the acquiring company or cash or a combination of shares and cash in exchange for the surrender of their stocks. The word merger refers to negotiations between friendly parties who arrive at a mutually agreeable decision. *Tender offers* usually convey that one firm or person is making an offer directly to the shareholders to sell their shares. Tender offers can be either friendly or unfriendly. Friendly tender offers occur when the the directors of the target company endorse the tender offer proposal; unfriendly tender offers do not have the same agreement from the target firm. An *acquisition* is an alternative term to tender offer. Mergers and acquisitions (tender offers) are two forms of *takeovers*. It is hence appropriate to refer to these activities interchangeably as takeovers or M&As or M&A activity.

It is widely accepted by economists that mergers can be grouped based on whether they take place at the same level of economic activity. From this point of view, mergers may be horizontal, vertical, or conglomerate. A *horizontal merger* involves two firms operating and competing in the same kind of business activity. Horizontal mergers are regulated by the government for their potential negative effect on competition as they could create monopoly power and gain monopoly profit. *Vertical mergers* occur in different stages of production operation. For

instance, in the oil industry, it involves exploration, production, refining and marketing to the ultimate consumer. The reasons why firms might want to be vertically integrated include technological economies that result in cost reduction or enhanced information efficiency. *Conglomerate mergers* involve firms in unrelated types of business activities. Conglomerate mergers can be further sub-categorized into financial conglomerates and managerial conglomerates. *Financial conglomerates* develop financial planning business and control systems for groups of segments that may be otherwise unrelated from a business point of view. Financial conglomerates undertake strategic planning in terms of assigning funds to segments according to performance of the segments. The operating decisions, however, are not delivered by financial conglomerates. *Managerial conglomerates* carry the attributes of financial conglomerates still further by providing managerial advice and interactions on decision, which enhance the potential for performance improvement.

In a tender offer, the bidder typically seeks the approval of the management in the target company, and, in the mean time, makes an offer directly to the shareholders in the target firm. Approval by over 50% of the shareholders in the target firm gives control to the bidder. In a merger, traditional legal doctrine held that the minority must agree to the terms negotiated. In a tender offer, the offer is extended to individual shareholders so that the management and the board of directors can be bypassed.

Tender offers may be conditional or unconditional. For example, the offer can be contingent on obtaining 50% of the shares of the target. Tender offers possibly will be restricted or unrestricted with respect to some classes of equity holders.

By law, shareholders of a target firm have a 20-day waiting period before they decide whether to accept the tender offer or not. The law also requires that when a new tender offer is made, the stockholders of the target company must have 10 business days to consider that new offer.

Tender offers can be made through a so-called two-tier format. The first tier receives an offer with superior terms, which typically consists of pure cash payment, and the second tier can only receive a lower price or less favorable terms. The second tier is often paid in securities such as debt or equity of the bidder. A variation of two-tier offer is the “three-piece-suit”. The three steps include (1) an initial toehold, (2) a tender offer to obtain control, and (3) after control and a majority of shareholders have tendered, a freeze-out purchase of the minority shareholders.

### 1.1.2 Motives for M&As

BERKOVITCH AND NARAYANAN (1993) summarize three major motives for M&As that are widely discussed in the literature: synergy, agency, and hubris. The synergy motive indicates that economic gains can result from merging the resources of the two firms. Therefore, the return to both bidder and target shareholders would be positive. The agency motive suggests that managers of the bidder firm, who are not acting to maximize shareholders' value, enhance their own welfare at the expense of shareholders via a takeover deal. It should be noted that the target firm, although being identified by the bidder management as a way to increase its own welfare, would gain a positive return from the deal. The gains to the bidder firm, however, as well as the total gains of the deal, should both be negative. The hubris theory claims that M&As are the result of mistakes in evaluating the target firm. Bidder firm engages in takeovers simply because they overestimate the benefit of the deal, which in fact consists of no synergy. The takeover deal motivated by hubris can be described as a zero-sum game where the total gain is zero and the bidder firm overpays due to its over-optimism. The payment to the target represents a value transfer between the target and the bidder.

WESTON ET AL. (1998) provides a comprehensive framework to examine the motives for mergers and acquisitions. I list the main points as below:

1. Total value increase
  - (a) Efficiency increase
  - (b) Operating synergy
  - (c) Diversification
  - (d) Financial synergy
  - (e) Strategic realignments
  - (f) The Q-ratio
  - (g) Information
2. Hubris - bidder overpays for target
3. Agency - managers make value-decreasing takeovers to increase size of firm
4. Redistribution
  - (a) Taxes - redistribution from government
  - (b) Market power - redistribution from consumers



- (c) Redistribution from bondholders
- (d) Labor - wage adjustments
- (e) Pension reversions

### **Total value increased**

#### **Efficiency increase**

If a relatively inefficient target is successfully taken over by a relatively efficient bidder, value can be increased by improving the efficiency of the target. Value can also be increased when management of either side can improve the efficiency of the combined firm. Combining firms may achieve better capital allocation and better utilization of investments. A discrepancy in efficiency would most likely to be a factor in mergers between firms in related industries.

#### **Operating synergy**

The theory based on operating synergy points out that economies of scale or scope are the major factors driving takeover deals. It assumes the firms are operating at levels of activity that fall short of reaching the potentials for economies of scale or scope before the merger. Through the deal, the complementarity of capabilities can be achieved. For example, merging a firm strong in research and development but weak in marketing with a firm strong in marketing but weak in research and development is expected to provide operating synergy.

#### **Diversification**

Compared to shareholders who can diversify across industries and firms in the capital market, employees of the firm have a very limited opportunity to diversity their labor income sources. Managers and other employees are at greater risk especially when the industry where their firm operates declines, as their firm-specific human capital is not transferable or only transferable within the industry. As a consequence, firms may diversify, i.e., engage in a cross-industry takeover, to encourage firm-specific human capital investments that make their employees more valuable and productive.

#### **Financial synergy**

Financial synergy theory considers complementarities between merging firms, not in management capabilities, but in matching the availability of investment oppor-

tunities and internal cash flows. It is widely accepted that internal financing costs less than external financing. Firms operating in a declining industry normally possess large internal cash flows but small investment opportunities. Firms in a growing industry normally have the opposite position and may have needs for additional financing. Combining the two may result in both a better allocation of capital and lower costs of internal financing.

Another possible financial benefit generated from a takeover deal comes from a greater debt capacity. The debt capacity of the combined firm can be greater than the sum of the two firms' individual capacity prior to the deal, and this therefore provides an extra tax saving. The studies LELAND AND SKARABOT (2003) and LELAND (2005) attempt to explore this sort of benefit. They both state that pure financial synergy, i.e., a better capital structure to enhance the total value of combined firm, can be captured via a merger. FLUCK AND LYNCH (1999) also discuss financial synergy driven takeovers.

### **Strategic realignments**

Takeovers motivated by strategic realignments have an emphasis on seeking new management skills to develop capabilities of the firm to adapt to changing environments. Acquiring external capabilities is believed to be quicker and safer than developing internal capabilities.

### **The Q-ratio**

The Q-ratio is the ratio of the market value of the firm's securities to replacement costs of its assets. If Q-ratio of the firm is substantially less than 1, it provides an opportunity for firms operating in the same industry to acquire additional capacity in a cheaper way by purchasing all the shares of the firm than directly purchasing the same amount of assets in the market. DONG ET AL. (2006) examine empirically Q theories of takeovers in different time periods and conclude the evidence is broadly consistent with hypotheses.

### **Information**

The information theory attempts to explain why target shares seem to be revalued upward on a long-term base in a tender offer regardless of being successful or not, found by BRADLEY (1980) and DODD AND RUBACK (1977). The information that target shares are undervalued is disseminated by the tender offer and therefore it is not necessary for the target firm to take any particular action to

cause revaluation. Alternatively, a tender offer might inspire target firm management to take some value-enhancing actions and therefore results in an upward revaluation of target firm shares.

### **Hubris**

R.ROLL (1986) was the first to suggest that the optimism and overconfidence can play a crucial role in the takeover decision with his "hubris" theory of corporate takeovers. He argues that takeovers are the result of the winner's curse that causes bidders to be overoptimistic with the target value and consequently overpay. Assuming strong market efficiency in all markets, the market price of the target firm already reflects its full value and therefore the higher valuation of the bidder in excess of the target's true economic value results from its excessive self-confidence (pride, arrogance).

The hubris/overconfidence helps interpret the evidence on merger announcement effects that while target firms earn positive abnormal returns through the deal, bidding firms, on the other hand, are found to realize negative to zero abnormal returns (see for example, JENSEN AND R.RUBACK (1983)).

As a recent contribution to this area, DOUKAS AND PETMEZAS (2007) take a look at the UK market for more evidence for overconfidence and its consequence in corporate takeover deals. A distinct feature of the UK data (1980-2004) is that 91% of takeovers are associated with private targets and therefore the decision to acquire is more likely to be based on managers' beliefs about potential synergies due to the lack of the public information. The evidence provides further support to the prediction that overconfident managers fail to generate superior abnormal returns relative to those created by "rational" managers.

### **Agency issue**

It is worth distinguishing two different views in terms of agency problems' application to M&A activities. JENSEN AND W.MECKLING (1976) discuss the implications of agency problems. One of the agency problems results from a conflict of interest between managers, who own only a fraction of the ownership shares of the firm, and other equity holders. The partial ownership structure may incentivize managers to pursue private welfare as the cost of other shareholders. There are a number of organizational and market mechanisms, such as compensation arrangements for managers (FAMA (1980)) or stock market as the function of external monitoring device (FAMA AND JENSEN (1983)), which attempt to mitigate or control agency problems. Nevertheless, when all these mechanisms

are not sufficient to control agency problems, takeovers are believed to act as an external control device of last choice (MANNE (1965)). MANNE (1965) also states that a firm's managers are exposed to a threat of takeover if their performance lags behind due to agency problems. On the other hand, some scholars consider M&As as a direct outcome of agency problems rather than a solution. MUELLER (1969) argues that managers, whose compensation is assumed to be a function of the size of the firm, have the incentive to increase the size of their firms, which can be realized through a takeover deal. Free cash flow hypothesis (JENSEN (1986)) provides a combined view of the two points aforementioned. It shows how takeovers are both evidence of conflicts of interest between shareholders and managers, and a solution to the problem. Free cash flow theory predicts that mergers are more likely to destroy value, rather than create value. The theory believes that free cash flows should be paid out to shareholders, reducing the power of management and subject managers to the scrutiny of public capital markets more frequently, which in turn mitigates agency problems. However, takeover is an alternative way for managers to spend cash instead of paying it out to shareholders and will consequently lead to inefficiency. On the other hand, leveraged buyouts (LBO), which normally involve a large amount of debt issuing, provide a device of bonding the managers' promise to pay out future cash flows to shareholders, which hence alleviates the impact of agency problems.

## **Redistribution**

### **Tax gains**

Tax represents a form of redistribution from the government or public at large. Takeovers may be motivated by the carryover of net operating losses and tax credits, stepped-up asset basis, and the substitution of capital gains for ordinary income. Intimidating inheritance taxes may also drive the sale of privately held firms with aging owners. However, it should be noted that tax gains are likely to be a reinforcing reason rather than a major motive in a sound takeover.

### **Market power**

Mergers might be driven by increased concentration leading to collusion and monopoly effects. There is evidence that concentration is the result of severe and continuing competition, which causes the composition of the leading firms to change over time.

### **Redistribution from bondholders**

MCDANIEL (1986) and WARGA AND WELCH (1993) find that in leveraged buy-outs in which debt is intensively used, there is evidence of negative impacts on bondholders. If the debt level is increased by very high orders of magnitude, the downgrading of corporate bond of bidder firm should be resulted for some cases and represent a value redistribution from bondholders to shareholders.

### **Redistribution from labor**

SHLEIFER AND VISHNY (1988) draw the attention to redistribution from labor to shareholders. They argue that the trust between shareholders and labor can be breached in various circumstances as a consequence of a takeover. For example, the investment made by employees to develop firm-specific skills are not paid their full value when previous labor contracts are broken by the new control group.

### **Pension fund reversions**

PONTIFF ET AL. (1990) examine another aspect of the breach of trust. They find that there is evidence of executing pension asset reversions in both hostile and friendly takeovers. Terminations or returning excess funding to the firm benefit shareholders, and workers lose.

Apart from the framework provided by WESTON ET AL. (1998), in a recent work by SHLEIFER AND VISHNY (2003), a market misvaluation problem has been presented. It states that a firm with overvalued equity might become the bidder of a takeover deal, and the one with undervalued or relatively less overvalued equity becomes takeover targets. FRIEDMAN (2004) presents similar idea and also provides empirical evidence.

### **1.1.3 The performance of M&As**

Both practitioners and researchers would be interested in the following questions: Do M&A activities really increase value? If value is created, is it maintained? Is value maintained merely in the short term or for a long term? Attempting to provide answers for these questions, a number of empirical studies focus on examining the performance of M&As. I review the major discoveries in the following contents.

### Successful takeovers

JENSEN AND R.RUBACK (1983) review 13 studies with sample data ending mostly in the late 1970s. Six of the studies are on mergers and seven on tender offers. They show a 30% positive return to target shareholders in successful tender offers and 20% in successful mergers. JARRELL ET AL. (1988) summarizes results for 663 successful tender offers from 1962 to 1985 and observe positive returns to targets within ranging from 19% to 35%. BRADLEY ET AL. (1988) examine the period July 1963 to December 1984. The sample consists of 236 successful tender offers. The returns to targets are between 19% and 35%, which vary in different subperiods. A more recent study with a 1,814 sample covering the years 1975 through 1991, produced by SCHWERT (1996), indicates targets in successful tender offers achieve 35% positive return. It is tempting to conclude that targets in successful tender offers or mergers earn substantial positive returns. The only issue is the magnitude.

With respect to returns to bidder firms, JENSEN AND R.RUBACK (1983) conclude that the excess returns to bidder firms in successful tender offers are positive 4% and zero in mergers. Positive but insignificant returns to bidder firms in successful tender offers are also found by JARRELL ET AL. (1988). BRADLEY ET AL. (1988) have similar results for tender offers. For 1960s, 1970s and 1980s, the returns to bidding firms are respectively around 4%, 1.3% and -3%. The data for the 1960s and the 1980s is significant at the 1% level. SCHWERT (1996) suggests that on average the abnormal returns to bidders are not significantly different from zero. In general, the excess returns to bidders in successful takeovers are close to zero, which is consistent with the expectation under perfectly competitive market.

WESTON ET AL. (1998) draw a conclusion that evidence suggests returns to target firms increase over the decades as a result of tighten government regulation and increasingly sophisticated defensive strategies developed by target firms. On the other side, returns to bidding firms suffer from the same factors over time. As a result, an opposite pattern for bidder's returns has been observed during the same period.

### Unsuccessful takeovers

JENSEN AND R.RUBACK (1983) discover that for unsuccessful tender offers both bidders and targets experience negative excess returns of modest size, but neither statistically significant. In terms of mergers, target firms also experience negative but statistically insignificant returns, while the bidder firms experience statistically significant negative returns. BRADLEY ET AL. (1983) show that

unsuccessful targets (the targets not taken over within the five-year period) experience positive returns at the beginning (within 60 days). After two years, their returns become negative and drift between a range of -5 to 10% during the subsequent three years. On the other hand, the experiences of unsuccessful bidding firms vary greatly, depending on the initial response in the announcement period and subsequent moves of bidders. WESTON ET AL. (1998) suggest the penalization for losing value-creating opportunities might help explain the loss for takeovers participants in unsuccessful deals.

### **Total returns**

BERKOVITCH AND NARAYANAN (1993) investigate a question of critical importance: in aggregate, do M&As create value? They develop tests to distinguish deals driven by three different motives: synergy, agency and hubris. They test 330 tender offers during 1963-1966 and find that about 75 % of the deals in the sample yield positive total return. The correlation between target and total gains is positive when the total gains are positive and is negative when the total gains are negative. They conclude that total returns are positive for most deals as synergy seems to be the dominant driving force in takeovers.

HOUSTON AND RYNGAERT (1994) examine 153 mergers among large banks during the period 1985-1991 and point out that the average total return to a completed bank merger is slightly positive but not significantly different from zero. However, the total merger returns for later years in their sample are significantly positive.

A recent study by MULHERIN AND BOONE (2000) examines 1,305 acquisition and divestiture deals during the period of 1990-1999. They find that both acquisitions and divestitures create wealth measured by the combined share price reaction to the announcement, which supports the argument that synergy effect is one of the major drives for transactions. They also find that combined bidder and target returns are significantly correlated to the ratio of target to bidder value.

### **Post-takeover performance**

HEALY ET AL. (1992) study the post-takeover performance of the 50 largest U.S. mergers between 1979 and 1984 and conclude that the industry-adjusted post-takeovers show improvement from better management of the assets. They find that the event returns for the firms, on average, correctly forecast post-takeover performance.

Covering the firms smaller than in HEALY ET AL. (1992), AGRAWAL ET AL.

(1992) obtain the evidence that the shareholders of bidding firms actually experience a wealth loss of about 10% over the five years following the deals. Their returns are adjusted for size effects and for beta-weighted market returns.

WESTON ET AL. (1998) conclude that the results of post-takeover performance are sensitive to sample selection and measurement methodology. Some takeovers perform well, but others do not. They point out that since the industry-adjusted post-takeover performance obtained by HEALY ET AL. (1992) is positive, while the market or economy-wide adjustments result in negative returns, found by AGRAWAL ET AL. (1992), it might imply that takeovers take place mainly in industries where performance is inferior to the whole economy.

### **Competition effect**

BRADLEY ET AL. (1983) analyze the competition effect on M&A transactions by comparing the excess return in single bidder case with that in multiple bidder contests. The results are consistent with what one expects - bidders on average “pay too much” under the competition threat. Generally, the competition makes the excess returns to bidders nearly zero and benefits targets. BERKOVITCH AND NARAYANAN (1993) also examine the impact of competition among bidders and conclude that competition appear to aggravate agency problem and stimulate hubris.

FULLER ET AL. (2002) does not fall into any category discussed above. They look into shareholder returns for the same bidder who makes bid for five or more public, private and/or subsidiary targets within a short time period. They find that bidders experience positive gains when buying a private firm or subsidiary but losses when purchasing a public firm. Besides, returns to bidders are greater, the larger the target firms, and if bidder offers shares.

To summarize, empirical findings appear to support that M&A activities do create value, while a majority part of the value goes to the target.

#### **1.1.4 Valuation of takeover bid**

As discussed previously, it can be concluded that some M&A activities do generate a positive economic effect. Some deals, however, fail to do so. In some cases, bidders seem paying too much under competition, which is the major reason of M&A failures. Legally, target shareholders have a waiting period to evaluate the tender offer and this gives other potential bidders the chance to join the bidding. In a takeover contest, it is obvious that the bidder with the highest estimate of target value will be able to submit the highest offer and eventually win the



contest. Therefore, the valuation of a takeover deal is crucial to both theoretical and practical issues.

WESTON ET AL. (1998) provide summary to some widely accepted valuation approaches. It is commonsense that similar companies should be valued at similar prices. Based on this rationale, the comparable companies or comparable transactions approach values the M&A transactions straightforwardly. These methods are easy to apply and appeal the M&A participants. Moreover, it can be used to establish valuation relationships for a company that is not publicly traded. However, the weakness of this method is also apparent. One cannot always find similar transactions or similar companies, thus this method can only provide an approximate estimation.

Traditional corporate finance theory recommends to use the discounted cash flow (hereafter DCF) approach to value a project. In terms of M&A valuation, DCF method is still applicable. The spreadsheet and formulae approach fall into this category and reader may refer to WESTON ET AL. (1998) for more detail regarding these two methods. However, every method has its own weakness and DCF method is no exception.

Implementing the DCF method (both the formulae method and spreadsheet method) requires forecasting post-takeover cash flows and estimating a proper discount rate relative to these projected cash flows. The DCF approach provides a rational economic framework for valuing takeovers that the marketplace generally follows. However, the DCF method is highly sensitive to cash flows and discount rate estimation and is also highly sensitive to takeovers made for growth, profit, margin, and terminal value. In addition, a target company's future cash flows depend on the method of takeover payment and the purchase price, letting alone that discount rate is also in debate.

MYERS (1987) proposes a similar idea about the problems of DCF method. He states that applying DCF analysis correctly has to overcome human and organizational problems that bias cash flows and discount rates. Besides, it is less helpful in valuating businesses with substantial growth opportunities or intangible assets. The basic insufficiency of the DCF approach to capital budgeting is that it ignores management's ability to revise its original strategy. In other words, DCF method is unable to consider managerial flexibility while there are various kinds of management strategies under an uncertain environment, which should be reflected in an evaluation model. MYERS (1987) also discusses the problems of DCF method with respect to bridging today's investments with tomorrow's opportunities. Sometimes an investment may appear irrational today when viewed in isolation, for example, with a negative net present value (NPV).

However, it is still reasonable to undertake this investment in the future when market conditions turn favourable.

As a solution to above problems, real options analysis has been widely discussed in recent years. In the traditional DCF method, One forecasts the future and treat the forecast as if it were true. One uses a risk adjusted discount rate. Real options analysis (HOWELL ET AL. (2001)), however, shows that the future is unforeseeable; one knows only the market condition of today, and one uses the risk-free rate of discount. SCHWARTZ AND TRIGEORGIS (2001) remark that there are three major advantages using this risk-neutral framework to value investment projects in comparison with the traditional NPV method. First, it properly considers all the flexibilities that a project might have. Second, it uses all the information contained in market prices (e.g., futures prices) when they exist. Third, it makes use of the powerful analytical tools developed in contingent claims analysis to determine both the value of the investment project and its optimal operating policy. The real options literature will be discussed in section 1.2.

### 1.1.5 Takeover bidding theories

Once the valuation of a target is obtained, a bidder issues a tender offer to the shareholders of the target based on that valuation. Generally, the bidder determines its offer price by adding some takeover premiums to its target valuation. Where do takeover premiums come from and what is the critical premium that makes the takeover successful? Both of them are main issues involved in takeover bidding theories.

BERKOVITCH AND KHANNA (1991) develop a model of the takeover market, in which the bidder has a choice between two takeover mechanisms: mergers and tender offers. Tender offers are modelled as an English auction in which bidders arrive sequentially and compete for each other. Mergers, however, are modelled as bargaining process between the bidder and target management who negotiates for the target shareholders. BERKOVITCH AND KHANNA (1991) focus on the important distinction between these two takeover mechanisms in the content of information: more information is released when a public tender offer is made than when relative secret merger negotiations occur. They show that there exists a unique level in the range of possible synergy gains. Tender offers are only possible when the bidder is able to generate synergy greater than that threshold. Otherwise, the bidder will make only merger attempts. The results imply that target returns from tender offers should be higher than those from mergers, and should increase with potential competition. Bidder's returns, on the other hand, are negatively correlated with competition. Besides, the compensation package of target

management also has an impact on the choice of takeover mechanisms: to get tender offers, target shareholders must provide their managers golden parachutes. Many efforts have been made to describe tender offers. Two different theories have been discussed. One of them assumes that either there is only one target shareholder or all the shareholders behave as a unit to rule out any free-rider problem, and there are multiple bidders involved in the competition (e.g., BERKOVITCH AND KHANNA (1991), GIAMMARINO AND HEINKEL (1986), FISHMAN (1988), HIRSHLEIFER AND PNG (1989), DANIEL AND HIRSLEIFER (1995) and KHANNA (1997)). The other one bases on the assumption that there is only one bidder who provides the tender offer directly to a large number or even infinite number of independent shareholders (for example, GROSSMAN AND HART (1980), GROSSMAN AND HART (1981), SHLEIFER AND VISHNY (1986), BAGNOLI AND LIPMEN (1988), HIRSHLEIFER AND TITMAN (1990), HARRINGTON AND PROKOP (1993) and YILMAZ (1999)).

GIAMMARINO AND HEINKEL (1986) set up a model in which the target makes the decision to accept or reject the first bid, when rejection still allows a second bidder to enter. The model assumes a common synergy to both potential bidders and allows the bidders to make any number of bids. However, two bidders are not equally informed and therefore sometimes it causes the uninformed bidder to overbid.

FISHMAN (1988) constructs a two-bidder takeover bidding model in the context of asymmetric and costly information. The first bidder has the ability to affect the second bidder's decision by conveying the information from his initial bidding price. After observing the first bidder's initial price, the second bidder updates his valuation of the target, which affects his bidding strategy. Taking into account the second bidder's bidding strategy, the first bidder settles on his initial offer price. FISHMAN (1988) models this strategic interaction between two potential bidders. The first bidder can deter the second bidder from offering a pre-emptive high price that signals a high valuation or accommodate the competition by offering a lower price. The essence of FISHMAN (1988) is that "it is not the bid itself that preempts a second bidder, but rather the information conveyed by the bid". The model implies that the lower the cost of exploring the information to the second bidder, the more incentive the first bidder would have to initiate an auction.

HIRSHLEIFER AND PNG (1989) present a model based on a similar framework. They assume that first bidder makes either a pre-emptive bid or a lower bid that induces the second bidder to investigate and possibly compete. In stead of assuming that bidder may costlessly revise its bid as the price changes, they

assume that bidding is costly. Standard English auction is therefore incomplete as a model of takeover bidding. The price of target may not be the minimum of bidders' valuations less the bidding cost and the price of target need not be higher with competitive bidding than with a single pre-emptive bidding. They show that the expected price of the target may be higher when the first bidder makes a pre-emptive bid and therefore the regulatory and management policy efforts to encourage competition may seem inefficient in terms of both the expected takeover price and social welfare.

DANIEL AND HIRSLEIFER (1995) analyze the large jumps in corporate takeover bidding contest via a theory of sequential bidding. By modelling tender offer as costly information sequential bidding game, KHANNA (1997) also introduces the effect of management's resistance and agency costs. The aim of the first bidder's pre-emptive offer is to shorten the bidding length, exclude other bidders, and then decrease the expected value of the target. The target's resistance can be viewed as a mechanism to delay this effort and improve the target shareholder's value. However, resistance may also result in agency costs, which could destroy shareholder's value.

On the other hand, GROSSMAN AND HART (1980) have led to a large literature on the "free rider" problem in takeovers. This problem is based on atomistic stockholders. Every stockholder has his own rational expectations that none of their decisions can affect the outcome of takeover attempts. Therefore, each of them demands a post-takeover value for his share (otherwise, he would free ride on the improvement brought by the bidder). Thus, the bidder can never gain from a takeover and will not make any offer. Even if small stockholders are uncertain about the type of bidder, GROSSMAN AND HART (1981) conclude that "free rider" problem still persists. According to GROSSMAN AND HART (1980), exclusive device can be built into the corporate charter to overcome this problem. The idea is to exclude minority stockholders from enjoying the benefits of takeover.

A large numbers of papers have been written on analyzing the robustness of this free rider problem. SHLEIFER AND VISHNY (1986) and HIRSHLEIFER AND TITMAN (1990) point out that even if shareholders are atomistic, takeover bid is still profitable. It is because bidders can earn an excess return on its current holdings. However, both of them do not explain how bidders acquire the initial shares. BAGNOLI AND LIPMAN (1988) make a more realistic assumption regarding the pattern of target shareholders. They suggest that stockholders are finite and ownership concentrates on some pivotal stockholders who cannot free ride on the takeover. They conclude that takeovers can succeed without exclusionary de-

vices. HARRINGTON AND PROKOP (1993) explore the free rider problem within a dynamic framework, which means that bidders are allowed to make more than one bids. Therefore, anticipation of a more attractive offer in the future makes stockholders more inclined to reject the offer now, which decreases the expected profit from a takeover bid. They conclude that exclusionary devices may indeed be necessary even when ownership is highly concentrated.

In a static and infinite number of shareholders model, it seems that there are only two approaches to overcome the free rider problem: a conditional offer that often provides the private benefits for the bidder (GROSSMAN AND HART (1980)) and initial holdings (toehold) (SHLEIFER AND VISHNY (1986) and HIRSHLEIFER AND TITMAN (1990)). However, YILMAZ (1999) presents a new framework in which shareholders are uncertain about the motives of the bidder. A bidder makes an offer either because it can generate higher cash flows or to pursue high private benefits of control. Consequently, shareholders cannot simply free ride on the improvements by the takeover that could never happen. Therefore, takeover profits may be positive and free rider problem does not have to happen definitely.

A latest research (FAIRCHILD AND GARROPAULIN (2007)) investigates the difference in the corporate ownership structure between developed and developing economies. They develop a game-theoretic model attempting to explain the separation of cash flow rights and control rights after the mergers evidenced in Mexico market. They demonstrate that the risk-aversion and private benefits factor can play roles in explaining the separation. For example, they find that higher risk-aversion can induce an incumbent manager to wish to minimize his equity stake but in the same time, with high private benefits, he would wish to retain control.

### 1.1.6 Methods of payment

The payment in mergers and acquisitions deals can be made in various forms: a pure cash offer, a pure share offer or a combination of cash and share. “In a perfect market with symmetrically informed agency, the medium of exchange chosen to accomplish a corporate combination is economically irrelevant; the level and division of the merger-induced gains are the same whether the transaction is executed by means of an all-cash offer or by some combination of cash and securities of the combined firm.” (ECKBO ET AL. (1990)). However, most empirical findings seem to support that both takeovers and takeover-induced abnormal returns are systematically related to the payment form. The choice of payment method in mergers and acquisitions is a very difficult issue because it is impacted by a wider range of variables and empirical studies are often contradictory.

## Asymmetric information

MYERS AND MAJLUF (1984) put forth the existence of asymmetric information between deal participants and market investors and they argue the method of financing an investment conveys information. They suggest that shares are used to finance a new investment opportunity because these shares are overvalued. On the other hand, if debt is issued to finance a new project, it implies the management team believes its common shares are undervalued.

TRAVLOS (1987) confirms asymmetric information theory by analyzing data between 1972 and 1981. He shows that returns to target firms are significantly positive in the two-day announcement period and there are higher abnormal returns in cash deals than in share deals. For bidders, TRAVLOS (1987) finds that stock exchange bidding firms experience significant losses at the announcement of the deal. In terms of cash offers, bidding firms earn normal rates of return due to the highly competitive nature of takeover market.

HANSEN (1987) proposes a model analyzing information asymmetry and employing the adverse selection and Nash Bargaining Equilibrium theories. In HANSEN (1987), no mixed offers can be observed because bidders use expected costs of synergy to signal their value to the target. The main difference between a cash offer and a share offer is that the value of a share offer is dependent on takeover returns, while it is not the case in a cash offer. He argues that when a target firm knows its value better than a potential bidder, the bidder favors offering shares, which have desirable contingent-pricing characteristics, rather than cash. His analysis covers 106 deals for manufacturing and mining firms during the period 1976-1978.

FISHMAN (1989) focuses on the role of the medium of exchange in preempting competition. In this model, targets and bidders are asymmetrically informed and bidders' offers bring forth potential competition. He concludes that cash has the advantage to preempt competition by signaling a high valuation for the target firm, hence providing higher returns to target shareholders.

In BERKOVITCH AND NARAYANAN (1990), the role of payment forms (cash, equity and a combination of two) in competition among bidders and their effects on returns to shareholders are investigated. It is also one of the asymmetric information models attempting to examine the interaction between informed bidders with favorable private information about potential synergy and uninformed targets. Their results, consistent with empirical findings, imply higher returns to both bidding and target firm's shareholders in cash deals than in share deals. They further argue that a higher proportion of cash used in a mixed offer will

lead to higher abnormal returns to shareholders for both bidder and target.

CORNETT AND DE (1991), however, find results that contradict the asymmetric information models. They examine 132 inter-state bank mergers between 1982 and 1986 and show that bidder's shareholders have positive abnormal returns which are significant at the 1% level, in cash offers, share offers and mixed offers. Two explanations are presented: (1) the impact of asymmetric information in banking sector might not be as important as in non-banking sectors; and (2) a share offer, in banking deals, might present some good information such as efficiency in asset management.

Following FISHMAN (1989) and HANSEN (1987), ECKBO ET AL. (1990) provide a theoretical explanation for the relation between the mix of cash and shares in an offer and the bidder's private information about its value as well as the value of the synergy. They identify that the true post-deal value of the bidder firm is revealed to the target by the composition of the mixed offer, and this revealed value is increasing and convex in the amount of cash used in the offer. Interestingly, their own empirical work conducted in the same paper does not seem to support their model predictions by using 182 Canadian takeovers in the period of 1964-1982.

A more recent work (CORNU AND ISAKOV (2000)) provides the results which are consistent with the argument that cash offers signal a high-valuing bidder and deter competition. LINN AND SWITZER (2001) manage to find evidence for this proposition.

## **Taxation**

A number of authors point out that the impact of taxation cannot be ignored by analyzing the choice of the payment method. It is widely addressed that cash offers are considered as immediately taxable for the target's shareholders and therefore require a higher premium to offset the incremental tax payment. On the other hand, share offers are favored from target shareholders' perspective as they are generally non-taxable until capital appreciation on shares are realized. From bidders' point of view, a higher takeover premium from a cash offer will lead to higher amortization of the goodwill in post-takeover years (which is no longer a problem under current US GAAP). The subsequent earnings will be artificially brought down, resulting in lower tax payment. Bidder shareholders will therefore prefer a cash offer if tax benefits from the deal is superior to higher premium paid, while the management team of target firm might make the opposite choice if they want to avoid firm performance artificially decreasing from amortization. (BLACKBURN ET AL. (1997))

WANSLEY ET AL. (1983) cover the mergers for the period 1970-1978 and show that target firm's cumulative average residual for 41 days through the announcement date when the method of payment is cash is 33.53% as compared with 17.47% when shares are used. The cumulative average residual for mixed payment form is 11.77%, lower than either pure cash or pure share. WANSLEY ET AL. (1983) believe tax plays an important role in payment choice as cash transaction are taxable to target shareholders so higher premiums paid to compensate for taxes to be paid. In addition, they point out that when shares are used, the bidding firm must go through Securities and Exchange Commission registration and it is not necessary to do so for cash offers. Consequently, it takes longer time to complete a share offer deal. The longer time before the deal is finalized, the higher possibilities for target management to develop a defense and a higher possibility for a rival bidder to turn up. These actions can result in high prices, which, nevertheless, could be offset by even higher preemptive cash bids.

In a later research, HARRIS ET AL. (1987) show that cash offers produce higher abnormal returns for targets. They examine a large sample of 2,500 acquisitions that have taken place in the UK and the USA between 1955 and 1985. The results have shown that cash only and share only offers have been the most frequent payment methods used in M&As. They develop the theory called tax and transaction cost efficient: shareholders who care about the liability of paying capital gains taxes prefer share offers, while the others will accept cash. However, they also state that, although being a possible explanation, there is no clear evidence that capital gains taxation is the major drive for the choice of payment method.

HUANG AND WALKING (1987) find similar results: average cumulated abnormal returns for cash offers are 29.3%, while 14.4% in share offers and 23.3% in mixed offers. The attractiveness of cash offers is explained by the impact of taxation.

## **Management control**

The management control theory is first presented by HARRIS AND RAVIV (1988) and STULZ (1988) and it argues that managers are reluctant to give up control so that they prefer not to finance takeovers by issuing shares. AMIHUD ET AL. (1990) conduct an empirical work on the relation between corporate control and the method of financing a corporate takeover. Their sample consists of firms that appear in the 1980 list of Fortune 500 companies and that take over other firms during the years 1981-1983. They find that the higher the managerial ownership fraction of the bidding company, the larger the probability of takeovers financed by cash rather than by a share exchange, which supports the management control



theory.

A later article by MARTIN (1996) covering 846 takeovers completed during 1978-1988 in either New York Stock Exchange (NYSE) or American Stock Exchanges (AMEX) finds consistent evidence. He argues that that higher managerial ownership is negatively related to the likelihood of stock finance.

All the studies aforementioned focus on the incentive of bidding firm. GHOSH AND RULAND (1998) present a study investigating primarily on target corporations and the motivation of their managers to obtain influence in combined firms after takeovers. They select the 50 largest acquisitions for each year from 1981 through 1988 and their results provide support to the hypothesis that managers of target firms who own large percentages of shares in their firms prefer receiving shares in exchange for their ownership and top managers are therefore able to maintain their voting influence to retain their jobs. They suggest that target's managerial ownership is even more important than the bidders' managerial ownership in explaining the method of payment for takeovers.

YOOK ET AL. (1999) examine the relative importance of two mainstream theories working to explain the choice of payment form in a corporate takeover deal: information asymmetry and management control. Using the inside trading as the proxy to measure asymmetric information, they find significantly more inside selling by the management of bidding firms before stock offers relative to cash offers as managers believe that overvalued shares prices will drop after a share involved takeover deal. In terms of management control concerns, they find that bidders with large inside holdings are more likely to offer cash, which demonstrates their strong desire to avoid diluted control.

FACCIO AND MASULIS (2005) use the data from European takeovers in the 1997-2000 period. In their model, the bidding firm is facing a tradeoff between corporate control concerns of issuing equity and rising financial distress costs by issuing debt. In contrast, a seller faces a tradeoff between tax benefits of stock and the liquidity and risk-minimizing benefits of cash consideration. Their findings are also consistent with the management control theory as they find that corporate control incentives to choose cash are particularly strong when a bidder's controlling shareholder has an intermediate level of voting power in the range of 20-60% - a range where it is most vulnerable to a loss of control. The incentive to avoid issuing shares is even strengthened when target shareholders are highly concentrated, which acts as an additional threat to bidder's control power in the newly created firm.

## Other factors

Apart from information, taxation and management control factors that are widely discussed, researchers believe there are some other issues that will affect the choice of payment in takeovers such as the size of participating firms, investment opportunity, and business cycle, etc.

GRULLON ET AL. (1997) examine 146 bank mergers during the period of 1981-1990 and their findings support the size effect. They find that the smaller the bidder in relation to target bank and the higher the bidder's capital adequacy ratio, the more likely it is that the shares are chosen. Their findings are also in line with prior studies that target banks earn positive abnormal returns.

However, some other studies, such as MARTIN (1996) and GHOSH AND RULAND (1998), do not seem to support the size effect. They find no significant links between relative size of bidder and target and payment methods chosen in takeover transactions. GHOSH AND RULAND (1998) argue it is the result of negotiation between two sides: if target is relatively larger than bidder, it prefers shares financing to maintain their interest and influence in the merged company; however bidder favors cash payment to avoid diluting their existing ownership in the firm.

MARTIN (1996) also explores the relationship between the payment method and firm's investment opportunities and business cycle conditions. He uses Tobin's q-ratio, the ratio of the market value of a company's debt and equity to the current replacement cost of its assets, as the proxy for firm's growth opportunity. The firms with greater q-ratio are believed to possess higher incentive to invest for future growth. MARTIN (1996) states that bidding firms with greater growth opportunities are more likely to use shares as the payment method in takeovers as they require cash to fund their growth investments. Moreover, MARTIN (1996) sheds light on the free cash flow (or cash availability) proposition that if bidders have sufficient cash flows in hand but few profitable investment opportunities, they tend to finance the takeover deals by cash.

MARTIN (1996) collects changes in the Standard and Poor's 500, index changes in Moody's BAA bond yield, changes in the index of 11 leading economic indicators and changes in industrial production to describe the business cycle. He identifies that Standard and Poor's 500 is significantly positively related to share financing, which provides evidence for business cycle proposition.

ZHANG (2003) examines how various factors, such as relative size, return on equity, dividend payout ratio, and ownership and market-to-book value of both bidding and target firms, influence the payment method in takeovers deals. He collects data from 103 UK mergers and acquisitions in the period 1990 through

1999. His research confirms relative size effect and free cash flow theory. He also argues that better performance of bidding firm, measured by larger market-to-book value, favors the choice of shares. However, ZHANG (2003) finds no evidence for the management ownership hypothesis.

In a more recent work, SWIERINGA AND SCHAUTEN (2007) look for evidence from Dutch mergers and acquisitions market. Their final data sample consists of 227 mergers and acquisitions announced during the period 1996-2005 by public bidders from the Netherlands. They believe the following characteristics significantly affect the payment method in their sample: bidder's fraction of closely held shares, bidder's asset size, bidder's market-to-book ratio, relative deal size, intra-industry deals and asset acquisitions. SWIERINGA AND SCHAUTEN (2007) find no clear evidence for the management control theory.

As outlined above, a number of hypotheses have been advanced to explore the choice of payment methods. The literature review of this specific area will conclude with a list of hypotheses in relation to payment form choice and its consequential impact on the performance of the deal.

- ***The asymmetric information hypothesis:***

- The returns to both bidding and target firm's shareholders are higher in a cash deal than in a share deal. (TRAVLOS (1987), FISHMAN (1989), BERKOVITCH AND NARAYANAN (1990))
- A higher proportion of cash used in a mixed offer should result in a higher return to shareholders for both bidder and target. (BERKOVITCH AND NARAYANAN (1990))

- ***The tax implication hypothesis:*** A higher return to target shareholders must be given to offset the greater tax exposure for a cash offer. (WANSLEY ET AL. (1983), HARRIS ET AL. (1987), HUANG AND WALKING (1987) )

- ***The management control hypothesis:***

- The higher the managerial ownership fraction of the bidding company, the higher the chance that deal is finance by cash rather than shares. (HARRIS AND RAVIV (1988), STULZ (1988), AMIHUD ET AL. (1990), MARTIN (1996), FACCIO AND MASULIS (2005))
- The cash offer is favored when target firm managers own a significant proportion of shares in their firms. (GHOSH AND RULAND (1998))

- ***The relative size effect hypothesis:*** The smaller the bidder in relation to the target, the shares are more likely to be used. (GRULLON ET AL. (1997))
- ***The investment opportunities hypothesis:*** The greater growth opportunities for bidders, the more likely share financing is used. (MARTIN (1996))
- ***The free cash flow hypothesis:*** The more the free cash flows owned by bidding firms, it is more likely that cash is used. (MARTIN (1996))
- ***The target pre-deal performance hypothesis:*** If the pre-deal performance of the target firm is bad, the bidder tends not to keep the inefficient management of the target firm, giving rise to cash financing more preferred. (ZHANG (2003))
- ***The correlation hypothesis:*** The intra-industry deals are more likely to be financed with shares than cross-industry deals. (SWIERINGA AND SCHAUTEN (2007))
- ***The deal size hypothesis:*** The larger the size of deals, the more likely the deal is financed with shares. (SWIERINGA AND SCHAUTEN (2007))
- ***The bidder ownership structure hypothesis:*** Bidders with an intermediate fraction of closely held shares favor cash more than bidders with a relatively low or high ownership stake. (SWIERINGA AND SCHAUTEN (2007))

## 1.2 Real options

### 1.2.1 A new view of investment

“Economics defines investment as the act of incurring an immediate cost in the expectation of future rewards” (DIXIT AND PINDYCK (1994)). Most investment decisions share three important characteristics to different degrees. First, investment is partially or completely irreversible. In other words, the initial cost of investment is at least partially sunk. Second, there is uncertainty over the future returns from investment and even the investment costs. Third, there are flexibilities with the timing of investment decisions. One can always choose to postpone its investment and wait for new information that might favor investment decisions in the future.

Traditional corporate theory suggests a very straightforward three-step rule to determine how to make an investment. Firstly, work out the present value of expected stream of earnings the project will generate. Secondly, calculate the present value of the stream of investment costs associated with the project. The final step will be working out the difference between these present values. The difference is so called net present value (NPV) and the decision simply relies on the sign of NPV: if it is positive, make the investment; otherwise, abandon it.

The NPV rule is straightforward, although it still involves some difficult issues such as determining the proper risk-adjusted discount rate or estimating the expected profits. Essentially it has a very straightforward judging rule: yes or never, based on the sign of the NPV. Clearly, NPV rule fails to take into account one important characteristic of investment decision: the flexibility to delay it. In other words, NPV rule simply ignores an opportunity cost, which should be included as part of the total investment costs, arising from the fact that investors have the right to delay their investment decisions.

Since BLACK AND SCHOLES (1973) and MERTON (1973) develop the financial option pricing theory, more and more studies have been accomplished to use this idea on the “real” business investment decision. For example, a company with an opportunity to invest in a project is like a financial call option: it has the right but not the obligation to buy an asset in the future. Since the investment is irreversible, once the company makes this investment, it “exercises” its call option. In other words, when a company makes this irreversible investment, it gives up its option value. Therefore, the option value should be incorporated into the whole project cost as an opportunity cost. Once this analogy has been set up, the generalized option pricing theory provides us a powerful tool to evaluate this opportunity cost quantitatively. In addition to the valuation, the investment timing can also be determined endogenously via a real options analysis.

### 1.2.2 Mathematical background

Before proceeding to the review of real options models, I intend to provide a brief introduction to the mathematical tools that are widely used in real options analysis. DIXIT AND PINDYCK (1994) provides a systematic description of applying a real options approach to capital investment decisions of firms. Two mathematical techniques are applied to evaluate a real options embedded project: dynamic programming and contingent claims analysis. In fact, they are closely related to each other and lead to the same outcome in many cases. However, they rely on different assumptions concerning the financial market and discount rate.

Dynamic programming breaks a whole sequence of decisions into two components: an immediate decision and a valuation function (continuation value) that incorporates the consequences of all subsequent decisions, starting with the position that results from the immediate decision. The optimal action is the one that maximizes the sum of immediate profit and continuation value. For a finite timing horizon, the very last decision can be found using static optimization methods and one can work backwards all the way to the initial decision. If the time horizon is infinite, what might seem like an even more complicated calculation is simplified by its recursive nature: each decision leads to another problem that looks exactly like the original one. Sometimes, it is even possible to obtain an analytical solution.

Contingent claims analysis bases on the economy that has the market for quite a rich menu of assets of all kinds. It is always possible to find some portfolios of traded assets (even if they are not directly traded, one can obtain an implicit value for them by relating them to other assets) to replicate the pattern of rate of return from the investment project. In order to avoid any arbitrage opportunities, the value of the investment project must equal the value of the replication portfolio. Therefore, investment project value can be derived from the market traded asset value. Once the value of investment opportunity is obtained, the timing of investment that achieves this value will be found consequently.

It will be proved useful to have a brief discussion of stochastic process that serves as a fundamental building block for all the models developed in the thesis. I will start from a Wiener process - also called a Brownian motion and then move to geometric Brownian motion, which is most widely accepted in the financial investment literature to describe the movement of share prices.

According to DIXIT AND PINDYCK (1994), A Wiener process is a continuous-time stochastic process with three important properties. One, it is a Markov process. Markov process essentially means that the probability distribution for all future values of the process depends only on its current value, and is independent of any past values of the process or any other current information. It means, to make a best forecast of its future value, the only information that is useful is its current price. Two, the Wiener process has independent increments. The probability distribution for the change in the process over any tiny interval is independent of any other time interval. Three, changes in the process over any finite interval of time have a normal distribution, with a variance that increases linearly with the time interval.

The Markov property of the Wiener process makes it suitable to describe some of the properties of stock prices. In a weak form of market efficiency, public

information should be incorporated in the current price of the stock immediately, therefore any past pattern of prices can be deemed as useless information to predict future value. This characteristic of share prices is perfectly in line with the Markov property. However, as share prices will never fall below zero, the normal distribution is ineffective in describing this feature. Alternatively, a geometric Brownian motion assumes that changes in stock prices are lognormally distributed, i.e., the changes in the logarithm of stock prices are normally distributed.

A geometric Brownian motion can be described by the following stochastic differential equation:

$$(1.1) \quad dx = \mu x dt + \sigma x dz,$$

or

$$(1.2) \quad \frac{dx}{x} = \mu dt + \sigma dz.$$

In the above equations,  $\mu$  is called the drift/expected growth parameter and  $\sigma$  the diffusion/variance parameter. The percentage changes in  $x$ ,  $dx/x$ , are normally distributed. Since these are changes in the natural logarithm of  $x$ , absolute changes in  $x$ ,  $dx$ , are lognormally distributed.

If the initial value (the value at the current time)  $x(0)$  is given by  $x_0$  and the value at time  $t$  by  $x(t)$ , the geometric Brownian motion indicated by equation (1.1) or (1.2) has an analytical solution:

$$(1.3) \quad x(t) = x_0 e^{((\mu - \sigma^2/2)t + \sigma z(t))}.$$

It can be further shown that the expected value of  $x(t)$  at time  $t$ , is given by

$$(1.4) \quad \mathcal{E}[x(t)] = x_0 e^{\mu t},$$

and the variance of  $x(t)$  is given by

$$(1.5) \quad \mathcal{V}[x(t)] = x_0^2 e^{2\mu t} (e^{\sigma^2 t} - 1).$$

Given that the discount rate  $r$  exceeds the drift (growth) rate  $\mu$ , the present value of a profit flow that follows a geometric Brownian motion can be given by

$$(1.6) \quad \mathcal{E} \left[ \int_0^\infty x(t) e^{-rt} dt \right] = \int_0^\infty x_0 e^{-(r-\mu)t} dt = \frac{x_0}{r - \mu}.$$

Another technique being very useful for real options analysis is Ito's Lemma. The stochastic processes have a part of which value is random over time, therefore in

terms of ordinary rules of calculus, these processes do not have a time derivative. Fortunately, Ito's Lemma allows us to differentiate and integrate functions of stochastic processes and furthermore conduct analysis based on it.

I only provide a statement of the lemma here and readers who have interests can refer to DIXIT AND PINDYCK (1994), NEFTCI (2000) and ØKSENDAL (2005) for proof and other detail.

Let  $x(t)$  be an stochastic process described by

$$(1.7) \quad dx = a(x, t)dt + b(x, t)dz,$$

where  $dz$  is the increment of a Wiener process, and  $a(x, t)$  and  $b(x, t)$  are known and non-random functions. The above process is well known as Ito or generalized Wiener process.

Consider a function  $F(x, t)$  that is at least twice differentiable in  $x$  and once in  $t$ . The total differential of this function,  $dF$ , suggested by Ito's Lemma, is given by

$$(1.8) \quad dF = \left[ \frac{\partial F}{\partial t} + a(x, t) \frac{\partial F}{\partial x} + \frac{1}{2} b^2(x, t) \frac{\partial^2 F}{\partial x^2} \right] dt + b(x, t) \frac{\partial F}{\partial x} dz.$$

Function  $F$  can involve several Ito processes. For example, suppose that  $F = F(x_1, \dots, x_m, t)$  is a function of time and of the  $m$  Ito processes  $x_1, \dots, x_m$ , where

$$(1.9) \quad dx_i = a_i(x_1, \dots, x_m, t)dt + b_i(x_1, \dots, x_m, t)dz_i, \quad i = 1, \dots, m.$$

The correlation coefficient between any two Wiener processes are given by  $\rho_{i,j}$ , which satisfies that  $\mathcal{E}(dz_i dz_j) = \rho_{i,j} dt$ . Ito Lemma provides the total differential  $dF$  by

$$(1.10) \quad \begin{aligned} dF = & \left[ \frac{\partial F}{\partial t} + \sum_i a_i(x_1, \dots, t) \frac{\partial F}{\partial x_i} + \frac{1}{2} \sum_i b_i^2(x_1, \dots, t) \frac{\partial^2 F}{\partial x_i^2} \right. \\ & + \frac{1}{2} \sum_{i \neq j} \rho_{i,j} b_i(x_1, \dots, t) b_j(x_1, \dots, t) \frac{\partial^2 F}{\partial x_i \partial x_j} \left. \right] dt \\ & + \sum_i b_i(x_1, \dots, t) \frac{\partial F}{\partial x_i} dz_i. \end{aligned}$$

In the following, I will present a model originally developed by McDONALD AND SIEGEL (1986) and further discussed by DIXIT AND PINDYCK (1994), to illustrate the basic techniques involved in a standard real options analysis.

The model considers a firm with an opportunity to invest in a single project. The cost of the investment, denoted by  $X$ , is known and fixed. However, the value of the project, denoted by  $S$ , follows a geometric Brownian motion:

$$(1.11) \quad dS = \mu S dt + \sigma S dz,$$



where  $\mu$  and  $\sigma$  are respectively drift and diffusion (variance) parameter. The term  $dz$  stands for a standard Brownian motion or Wiener process.

The firm's investment opportunity is therefore equivalent to a perpetual call option - the right but not the obligation to incur the investment cost,  $X$ , to achieve the outcome of the investment,  $S$ . The timing to exercise such a real investment option is when the investment will be conducted. As aforementioned, the optimal investment rule can be derived in two ways: dynamic programming or contingent claim analysis. With the rationale of the two methods behind us, the more technical detail will be covered as follows and it will allow us to compare these two approaches.

I will start from **dynamic programming**. Let  $F(S)$  be the value of the firm's option to invest. It yields no cash flows up to time  $T$  when the investment is undertaken. Therefore the only return from holding this investment opportunity is its capital appreciation. Hence, the Bellman equation in the continuation region, i.e., the value of  $S$  for which it is not optimal to invest, is expressed by

$$(1.12) \quad rFdt = \mathcal{E}(dF),$$

where  $r$  is a proper discount rate. The above equation indicates that over time interval  $dt$ , total expected return on the investment opportunity,  $rFdt$ , should have the same value as its expected rate of capital appreciation.

Ito's lemma gives the value of  $dF$ :

$$(1.13) \quad dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial S}dS + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2.$$

As the investment decision is irrelevant to calendar time, the value of  $\frac{dF}{dt}$  should be zero. Let  $F'(S)$  and  $F''(S)$  denote  $\frac{\partial F}{\partial S}$  and  $\frac{\partial^2 F}{\partial S^2}$  respectively, and the above equation becomes:

$$(1.14) \quad dF = F'(S)dS + \frac{1}{2}F''(S)(dS)^2.$$

Noting that  $\mathcal{E}[dz] = 0$  and  $\mathcal{E}[(dz)^2] = dt$ , the expected value is given by

$$(1.15) \quad \mathcal{E}[dS] = \mu Sdt,$$

$$(1.16) \quad \mathcal{E}[(dS)^2] = \sigma^2 S^2 dt.$$

Hence equation (1.12) becomes

$$(1.17) \quad \frac{1}{2}\sigma^2 S^2 F''(S) + \mu S F'(S) - rF = 0.$$

In addition, the value of  $F(S)$  must satisfy the following boundary conditions:

$$(1.18) \quad F(0) = 0,$$

$$(1.19) \quad F(S^*) = S^* - X,$$

$$(1.20) \quad F'(S^*) = 1,$$

where  $S^*$  represents the value of  $S$  upon the time of exercise.

Condition (1.18) arises from the observation that if  $S$  goes to zero, it will stay at zero with the implication of the stochastic process for  $S$ . As a result, the option to invest will be of no value when  $S = 0$ . Condition (1.19) is so-called “value-matching” condition that imposes equality between the value of the investment option and the payoff of the option upon exercise. Said differently, the value should be equal to immediate net payoff  $S^* - X$  upon exercise. Finally, condition (1.20) is the “smooth-pasting” condition. It ensures that investment occurs along the optimal path by requiring a continuity of slopes at the trigger threshold. If  $F(S)$  were not continuous and smooth at the critical exercise point  $S^*$ , one could do better by exercising at a different point.

DIXIT AND PINDYCK (1994) state that although the equation (1.17) is a second-order differential equation, there are three boundary conditions that must be justified. The reason being is that even if the position of the first boundary ( $S = 0$ ) is known, the position of the second boundary is not. In other words, the “free boundary”  $S^*$  must be determined as part of the solution, which consequently requires the third condition.

Equation (1.19) has another useful interpretation. Rearranging it gives  $S^* = I + F(S^*)$ . When the firm decides to invest, it obtains the project value  $S^*$ . In addition to the direct or tangible cost of investment  $X$ , the firm also forfeits its intangible opportunity cost  $F(S^*)$ , which is ignored by the traditional corporate finance theory, such as NPV rule.

Equation (1.19) is an ordinary second-order homogeneous differential equation and therefore should have a general solution expressed as a linear combination of any two independent solutions:

$$(1.21) \quad F(S) = AS^{\beta_1} + BS^{\beta_2},$$

where  $A$  and  $B$  are constants that are yet to be determined. The values of  $\beta_1$  and  $\beta_2$  are two roots of the quadratic equation

$$(1.22) \quad \frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0.$$

The two roots are

$$(1.23) \quad \beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1,$$

$$(1.24) \quad \beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0.$$

The boundary condition (1.18) implies that  $B$  must be zero, and the solution therefore must take the form

$$(1.25) \quad F(S^*) = AS^{*\beta_1}.$$

Substituting (1.25) into (1.19) and (1.20) gives:

$$(1.26) \quad AS^{*\beta_1} = S^* - X,$$

$$(1.27) \quad \beta_1 S^{*\beta_1-1} = 1.$$

Solving them provides:

$$(1.28) \quad S^* = \frac{\beta_1}{\beta_1 - 1} X,$$

$$(1.29) \quad A = \frac{(\beta_1 - 1)^{\beta_1-1}}{\beta_1^{\beta_1} X^{\beta_1-1}}.$$

Equation (1.28) specifically presents the optimal investment rule. Since  $\beta_1 > 1$ , I have  $\frac{\beta_1}{\beta_1-1} > 1$  and therefore  $S^* > X$ . It implies that the investment rule suggested by simple NPV analysis, which is, invest whenever the value of project exceeds its associated cost, is not valid anymore. The multiple  $\frac{\beta_1}{\beta_1-1}$  reflects the magnitude of optimal investment rule, which relies on the parameters  $\sigma$ ,  $\mu$  and  $r$ .

DIXIT AND PINDYCK (1994) examine the characteristics of  $\frac{\beta_1}{\beta_1-1}$  in more detail and find that as volatility  $\sigma$  increases,  $\beta_1$  decreases, and  $\frac{\beta_1}{\beta_1-1}$  should increase. The result reveals one of the most important implications from real options analysis: uncertainty delays investment. The greater the uncertainty (volatility) is involved in future project value, the larger is the gap between  $V^*$  and  $X$ , i.e., the larger the return the investor will require to give up its irreversible investment opportunity.

It is widely accepted that dynamic programming has its weakness in that it is based on an arbitrary and constant discount rate,  $r$ . To estimate the proper value of this discount rate, one has to account for investor's risk preference, which

varies over different investors in the market. *Contingent claims* method is an alternative method that avoids doing so and therefore is of many scholars' interest.

Contingent claims analysis, however, also requires one important assumption: the market must be sufficiently "complete" so that one can always find an asset or construct a dynamic portfolio of assets to replicate the uncertainty of underlying asset, e.g.,  $S$ . In other words, there must always exist an asset or a portfolio, the price of which is perfectly correlated with  $S$ .

Following DIXIT AND PINDYCK (1994), let  $\mathbb{S}$  be the price of an asset or a dynamic portfolio of assets perfectly correlated with  $S$ . Let  $\rho_{\mathbb{S}m}$  be the correlation of  $\mathbb{S}$  with the market portfolio. It is assumed that  $\mathbb{S}$  is perfectly correlated with  $S$ , hence  $\rho_{\mathbb{S}m} = \rho_{Sm}$ . It is further assumed that this asset or portfolio pays no dividend, therefore the entire return is from capital gains. Thus,  $\mathbb{S}$  should follow the stochastic process described by

$$(1.30) \quad d\mathbb{S} = \mu_{\mathbb{S}}\mathbb{S}dt + \sigma_{\mathbb{S}}\mathbb{S}dz.$$

To evaluate investment projects with contingent claims analysis, let  $F(S)$  denote the value of firm's option to invest and  $S$  follow a standard Brownian motion satisfying equation (1.11). The basic idea is to construct a risk-free portfolio, determine its expected rate of return, and equate that expected rate of return to the risk-free rate.

To construct a risk-free portfolio, one should hold the option to invest,  $F(S)$ , and, in the same time, go short  $n = F'(S)$  units of the project or equivalently, the asset or portfolio  $\mathbb{S}$  that is perfectly correlated with  $S$ . Let  $\Phi$  denote the value of this portfolio hence  $\Phi = F - F'(S)S$ . As  $F'(S)$  may change over time as  $S$  changes over time, the portfolio must be constructed dynamically, i.e., the composition of the portfolio will change over time. However, over a infinite small time interval  $dt$ , the value of  $n$  can be treated as fixed.

Considering  $\delta S$  as the dividend or cash flows paid out to the holder, the short position in the portfolio will have to make a payment of  $\delta S F'(S)$  to compensate the long side investors. Therefore, the total return from holding the portfolio over a short time interval  $dt$  is

$$(1.31) \quad dF - F'(S)dS - \delta S F'(S)dt,$$

where  $dF$  can be expanded using Ito's Lemma:

$$(1.32) \quad dF = F'(S)dS + \frac{1}{2}F''(S)(dS)^2.$$

Therefore the total return on the portfolio is given by

$$(1.33) \quad \frac{1}{2}F''(S)(dS)^2 - \delta SF'(S)dt.$$

Note that  $dS = \mu Sdt + \sigma Sdz$ , hence the return on the portfolio becomes

$$(1.34) \quad \frac{1}{2}\sigma^2 S^2 F''(S)dt - \delta SF'(S)dt.$$

To avoid arbitrage possibilities, this return must equal the risk-free rate:

$$(1.35) \quad \frac{1}{2}\sigma^2 S^2 F''(S)dt - \delta SF'(S)dt = r_f[F - F'(S)S]dt.$$

Eliminating  $dt$  on both sides and rearranging gives the following differential equation that  $F(S)$  must satisfy:

$$(1.36) \quad \frac{1}{2}\sigma^2 S^2 F''(S) - (r_f - \delta)SF'(S) - r_f F = 0$$

The equation obtained above is very similar to what one can obtain from a dynamic programming analysis (equation 1.17). In fact, under the assumption of risk neutrality, i.e., the discount rate  $r$  is equal to the risk-free rate  $r_f$ , the results obtained from the two methods are identical. I therefore omit the subsequent steps to derive the optimal investment rules through boundary conditions.

If the investment represents an opportunity of exchanging two stochastic assets, the above analysis has to be adjusted accordingly. Consider an investment opportunity that involves exchanging two stochastic assets  $S_1$  and  $S_2$ . For example,  $S_1$  might represent the stochastic future payoffs from the investment and  $S_2$  the random investment costs. The net payoff from the project clearly will be the difference of the two random assets  $S_1 - S_2$ , which in most cases should be stochastic as well. The exercise strategy therefore should maximize the expected value of this difference while the involvement of two random variables makes the analysis mathematically more difficult.

To be in line with standard real options literature,  $S_1$  and  $S_2$  follow the geometric Brownian motions:

$$(1.37) \quad dS_1 = \mu_2 S_1 dt + \sigma_1 S_1 dz_{S_1},$$

$$(1.38) \quad dS_2 = \mu_1 S_2 dt + \sigma_2 S_2 dz_{S_2}.$$

The correlation coefficient between the two sources of uncertainty is constant and equals to  $\rho$ :

$$(1.39) \quad \mathcal{E}[dz_{S_1} dz_{S_2}] = \rho dt.$$

Intuitively, one expects that the option of exchanging two stochastic variables  $S_1$  and  $S_2$  will be held when  $S_1$  is low or  $S_2$  is high. Such option should be exercised when  $S_1$  becomes sufficiently high for given  $S_2$ , or  $S_2$  becomes sufficiently low for given  $S_1$ . Following the intuition, one should expect the option to be exercised when the ratio of  $S_1$  to  $S_2$  exceeds a certain threshold, which is the optimal investment rule.

Let  $F(S_1, S_2)$  be the value of option to invest. Since the investment opportunity yields no cash flows up to time when the investment is executed, the only return from holding it is its capital appreciation. In the continuation region, the Bellman equation is:

$$(1.40) \quad rFdt = \mathcal{E}(dF),$$

where  $r$  is an appropriate discount rate or rate of expected return, and  $\mathcal{E}(dF)$  on the right hand side denotes the expected change of capital appreciation.

Expanding the right hand side of the above equation applying the general two-dimensional Ito's Lemma

$$(1.41) \quad dF = \frac{\partial F}{\partial t}dt + \sum_i \frac{\partial F}{\partial X_i}dX_i + \frac{1}{2} \sum_i \sum_j \frac{\partial^2 F}{\partial X_i \partial X_j}dX_i dX_j,$$

and noting that since this investment optimization problem is time-independent, there is no partial derivative with respect to time. Thus

$$(1.42) \quad \mathcal{E}(dF) = \mathcal{E} \left[ F_{S_1}dS_1 + F_{S_2}dS_2 + \frac{1}{2}F_{S_1S_1}(dS_1)^2 + \frac{1}{2}F_{S_2S_2}(dS_2)^2 + F_{S_1S_2}dS_1dS_2 \right],$$

where for any value function  $F$ ,  $F_i$  ( $i = S_1, S_2$ ) denotes the first-order partial derivative of  $F$  with respect to  $i$  and  $F_{ii}$  ( $i = S_1, S_2$ ) denotes the second-order partial derivative of  $F$  with respect to  $i$ .

Substituting for  $dV$  and  $dI$  (terms with  $dt$  of order greater than one can be ignored) provides:

$$(1.43) \quad \mathcal{E}(dS_1) = \mu_1 S_1 dt,$$

$$(1.44) \quad \mathcal{E}(dS_2) = \mu_2 S_2 dt,$$

$$(1.45) \quad \mathcal{E}(dS_1^2) = \sigma_1^2 S_1^2 dt,$$

$$(1.46) \quad \mathcal{E}(dS_2^2) = \sigma_2^2 S_2^2 dt,$$

$$(1.47) \quad \mathcal{E}(dS_1 dS_2) = \rho \sigma_1 \sigma_2 S_1 S_2 dt.$$

Substituting the above expected value into equation (1.40) and (1.42) leads to the partial differential equation that the investment option must follow:

$$(1.48) \quad F_{S_1}\mu_1 S_1 + F_{S_2}\mu_2 S_2 + \frac{1}{2}F_{S_1S_1}\sigma_1^2 S_1^2 + \frac{1}{2}F_{S_2S_2}\sigma_2^2 S_2^2 + F_{S_1S_2}\rho\sigma_1\sigma_2 S_1 S_2 - rF = 0.$$

Upon the time of exercise, the immediate payoff  $(S_1^* - S_2^*)$  from the option must equal the value of the option  $(F(S_1^*, S_2^*))$ . In addition, to avoid arbitrage, smooth-pasting conditions must be satisfied:

$$(1.49) \quad F(S_1^*, S_2^*) = S_1^* - S_2^*,$$

$$(1.50) \quad F_{S_1^*}(S_1^*, S_2^*) = 1,$$

$$(1.51) \quad F_{S_2^*}(S_1^*, S_2^*) = -1.$$

An additional boundary condition is given by requiring that, as the ratio of two exchanging assets  $(S_1/S_2)$  is too small, the ratio of option value to  $S_2$  tends to be zero. The idea behind it is not hard to explain. If  $S_1$  is too small compared with its cost  $S_2$ , investors should find the option very unattractive because the opportunity to invest would never be profitable.

$$(1.52) \quad \lim_{(S_1/S_2) \rightarrow 0} \frac{F(S_1, S_2)}{S_2} = 0$$

The feature that the boundary itself is an unknown makes problems of this kind quite difficult. Precisely, it involves solving free-boundary problems for elliptic partial differential equations, and analytical solutions are rarely available. However, the homogenous property of equation (1.48) makes it possible to reduce the dimension of the problem and obtain an analytical solution.

Intuitively, the optimal investment decision should depend on the ratio  $S_1/S_2$  rather than the absolute value of either  $S_1$  or  $S_2$ . A new variable is created:  $R \equiv S_1/S_2$ . Thus

$$(1.53) \quad F(S_1, S_2) = S_2 f(S_1/S_2) = S_2 f(R),$$

where  $f$  is the function to be determined.

Successive differentiation gives:

$$(1.54) \quad F_{S_1}(S_1, S_2) = f_R(R),$$

$$(1.55) \quad F_{S_2}(S_1, S_2) = f(R) - R f_R(R),$$

$$(1.56) \quad F_{S_1 S_1}(S_1, S_2) = f_{RR}(R)/S_2,$$

$$(1.57) \quad F_{S_1 S_2}(S_1, S_2) = -R f_{RR}(R)/S_2,$$

$$(1.58) \quad F_{S_2 S_2}(S_1, S_2) = R^2 f_{RR}(R)/S_2.$$

Substituting them into the partial differential equation (1.48) and the boundary conditions yields:

$$(1.59) \quad \frac{1}{2}(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)R^2 f_{RR}(R) + (\mu_1 - \mu_2)R f_R(R) - (r - \mu_2)f(R) = 0,$$

with boundary conditions:

$$(1.60) \quad f(R^*) = R^* - 1,$$

$$(1.61) \quad f_{R^*}(R^*) = 1,$$

$$(1.62) \quad \lim_{R \rightarrow 0} f(R) = 0.$$

A general solution for equation (A.33) is given by:

$$(1.63) \quad f(R) = AR^{\beta_1} + BR^{\beta_2},$$

where  $A$  and  $B$  are constants, and  $\beta_1$  and  $\beta_2$  are respectively positive and negative roots of the subsequent quadratic equation:

$$(1.64) \quad \frac{1}{2}(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)\beta(\beta - 1) + (\mu_1 - \mu_2)\beta - (r - \mu_2) = 0$$

Solving the above equation gives:

$$(1.65) \quad \beta_1 = \frac{1}{2} - \frac{\mu_1 - \mu_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} + \sqrt{\left(\frac{\mu_1 - \mu_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_2)}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}} > 1$$

$$(1.66) \quad \beta_2 = \frac{1}{2} - \frac{\mu_1 - \mu_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} - \sqrt{\left(\frac{\mu_1 - \mu_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_2)}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}} < 0$$

Boundary condition (A.36) indicates that  $B = 0$  and reduces the two solutions for  $\beta$  to a single one:

$$(1.67) \quad f(R^*) = AR^{*\beta_1}$$

Substituting (A.40) into (A.34) and (A.35) yields:

$$(1.68) \quad A(R^*)^{\beta_1} = R^* - 1$$

$$(1.69) \quad A\beta_1(R^*)^{\beta_1-1} = 1$$

Solving it for  $A$  and  $R^*$  gives the optimal investment rule:

$$(1.70) \quad \frac{S_1^*}{S_2^*} = R^* = \frac{\beta_1}{\beta_1 - 1}$$

Basic features of the result obtained above, suggested by DIXIT AND PINDYCK (1994) are laid out as follows: If either  $\sigma_1$  or  $\sigma_2$  increases,  $\beta_1$  will decrease, and the



multiple  $\frac{\beta_1}{\beta_1-1}$  will increase. Volatility delays the investment as greater uncertainty over future payoff enhances the incentive of decision makers to delay the decision and therefore wait for more information. However, the multiple  $\frac{\beta_1}{\beta_1-1}$  will decrease if the correlation coefficient between the two sources of uncertainty increase. It can be explained by a reducing uncertainty over the ratio through a greater correlation between changes in  $S_1$  and  $S_2$ , holding their variances fixed. In an interesting extreme case, if both  $S_1$  and  $S_2$  follow the same stochastic processes (different initial values are allowed) and are perfectly correlated ( $\rho = 1$ ), the option value to exchange  $S_1$  for  $S_2$  shrinks down to zero as there is actually no uncertainty involved exchanging two identical random assets and the investment rules will be subject to a standard NPV analysis, determined by the initial value of each random process.

### 1.2.3 Review of basic models

Leaving mathematical preliminaries behind us, I can now turn to reviewing some classic real options models. I also intend to provide necessary technique details for the studies most related to my research.

MYERS (1987) is the first author to extend the idea of financial options to real assets. He puts forth the thinking of discretionary investment opportunities as “growth options”. Following MYERS (1987), KESTER (2001) conceptually develops the growth option further more. TRIGEORGIS AND MASON (1987) provide another conceptual real options framework. Their work bases on the traditional NPV approaches to capital budgeting and puts forward the idea of expanded NPV, which includes the real options premium to justify managerial flexibility. Introducing the contingent claims analysis, they conceptually discuss a variety of real operating options: the option to defer investment, the option to expand, the option to contract and other types of operating options such as option to shut down operations temporarily, to switch use, or to abandon a project permanently when future market conditions turn out worse than originally expected. Other attempts to construct a conceptual framework for real options include DIXIT AND PINDYCK (2001) and TRIGEORGIS (1988).

These conceptual discussions work to provide a general conceptual framework for analyzing investment opportunities under the real options scenario. They describe various real options that might be embedded in investment opportunities and seek strategic planning with value maximization. These discussions serve as a conceptual basis for integrating the financial options methodology into real investment decisions.

BRENNAN AND SCHWARTZ (2001) consider a natural resource investment, the sector normally with a relative high volatility. They argue that traditional discounted cash flow approaches are subject to some severe limitations and problems such as the estimation of future output prices, and the determination of a proper discount rate. More importantly, they fail to account for the managerial flexibility that investors possess in terms of their future decisions. They suggest that a natural resource investment, such as a gold or copper mine, may be valued with the help of established financial option pricing approach pioneered by BLACK AND SCHOLES (1973) and MERTON (1973).

In a continuous time setting, DIXIT (1992) provides quantitative analysis on how much real options are worth. The optimal timing derived from value maximizing strategy provides a clear picture of how traditional net present value analysis is misleading. Specifically, he demonstrates why investors should invest until price rises substantially (two or three times with some reasonable parameters) above its long-run average cost and why it is not optimal to abandon a business immediately after it starts generating negative payoff.

KULATILAKA AND TRIGEORGIS (1994) consider a firm that has a choice to switch various operating modes to produce different outputs with uncertain demand. They suggest this flexibility to switch can be treated as a build-in option, which provides an extra value that cannot be ignored. The analysis discusses the impact of switching costs on the option value of switching and its consequent optimal exercise strategy. The hysteresis effect, an idea similar to what DIXIT (1992) points out, is also discovered through the analysis, i.e., even if short-term situation favors the decision of immediate switching, there is still the value of waiting. PINDYCK (1991) reviews some basic models of irreversible investment to illustrate the option-like characteristics of investment opportunities, and shows how to value the option value of deferring in a continuous time setting, via both option pricing (contingent claims) and dynamic programming analysis.

There are various real options embedded in real investment opportunities. I tend to follow TRIGEORGIS (2001) to group common real options and then give a brief review of some classic models in each category.

Option to defer or option to wait is probably one of the most well known real options. A typical deferring option brings forward the idea that management, who holds an investment opportunity associated with uncertain payoff in the future, can wait to see if earning from the investment justifies spending irreversible investment costs. The option to wait is analyzed by McDONALD AND SIEGEL (1986).

Some staging investment, as a series of sequences, creates the option to abandon

the project in midstream if new information favors it to do so. Essentially, each stage of the investment can be viewed as an option on the value of subsequent stages, which also represents an investment option. This kind of investment should be valued within a compound option framework and its embedded options are widely accepted as time to build options or sequential options. MAJD AND PINDYCK (1987) falls into this category.

If market conditions are more favorable than originally expected, investors should have the flexibility to expand the scale of production or accelerate resource utilization. If market evolves towards a converse way and the conditions are worse than expected, the investors would like to reduce operating scale or, when the market conditions are bad enough, temporarily shut down the operation and start up again in the future. These real options, suggested by TRIGEORGIS (2001), are generally treated as options to alter operating scale. They consist of options to expand, to contract and to shut down and restart. McDONALD AND SIEGEL (1985), TRIGEORGIS AND MASON (1987), PINDYCK (1988) and BRENNAN AND SCHWARTZ (2001) cover this category.

Sometimes, market conditions are severely unfavorable and to alter operating scale is no longer an efficient decision. Management may have to abandon current operations permanently to realize the resale value of initial investments. Option to abandon is discussed by MYERS AND MAJD (1990).

So far, it is assumed that investment costs (inputs) are certain and only future payoffs (outputs) are allowed to be uncertain. However, if the investment costs are also involved with some uncertainty, the investment execution becomes an exchange of two uncertain assets. MARGRABE (1978) provides a rigorous study on exchanging options (option to switch). Other studies regarding option to exchange are given by McDONALD AND SIEGEL (1985) and KULATILAKA AND TRIGEORGIS (1994) and DIXIT AND PINDYCK (1994)<sup>1</sup>.

The subsequent topic focuses on growth options. An early investment, such as research and development (R&D) or a strategic acquisition, is a prerequisite or initial platform that a chain of interrelated projects can build on. It essentially provides future growth opportunities that can follow up the initial movement. Real options of this kind are normally modelled as growth options and there are a number of papers fallen into this area such as TRIGEORGIS AND MASON (1987), TRIGEORGIS (1988), PINDYCK (1988) and KESTER (2001).

Several studies, such as TRIGEORGIS (1993) and BRENNAN AND SCHWARTZ (2001), attempt to handle a more complex situation. Some real-life projects involve a number of real options, whose value may be dependent on each other.

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<sup>1</sup> See Chapter 6.

The effort has been made to discover how various real options interact with each other and therefore the combined option value may differ from the sum of separate option values.

In the following, I will present a more detailed review of some representative research in each of the category mentioned earlier. MCDONALD AND SIEGEL (1986) present a continuous-time framework to analyze a firm's investment in a single project. They derive analytical formulas for the value and timing of waiting option to invest in an irreversible project, when both the value of the project and the cost of investing are stochastic (geometric Brownian motion). In addition to investment timing and the value of waiting, optimal scrapping timing and the issue of computing a correct discount rate are also covered by their research. MCDONALD AND SIEGEL (1986) furthermore explore the situation that the present value of net future cash flows takes a discrete jump to zero via modelling it as a mixed Poisson-Wiener process.

MAJD AND PINDYCK (1987) investigate the time-to-build options embedded in investment projects that have the following characteristics: first, investment costs occur sequentially over time; second, the rate of expenditures being made cannot exceed a certain maximum rate; third, the investment opportunities will not generate any cash flows until the entire sequence is completed. They show the optimal decision rule in each stage of the project and the project valuation through option pricing techniques. They also show dependence of investment decision on the maximum rate the expenditures must be made and how the level of risk brings forward a greater impact on the investment decisions due to the flexibility that firm has in making sequential investments.

MYERS AND MAJD (1990) suggest that to permanently abandon a project for its salvage value is analogous to an American put option on a dividend-paying asset. A numerical technique (finite-different method) is employed due to the absence of a closed-form solution. Essentially, they point out that the project life should not be fixed. It is subject to the decision to abandon the project, which is determined by a value maximizing strategy via options theory. Their propositions challenge the traditional discounted cash flow rules that routinely rely on a known project life and therefore have considerable practical implications.

MARGRABE (1978) develops a model for exchange options. Some common financial arrangements are equivalent to options to exchange one risky asset for another. This option is simultaneously a call option on one asset and a put option on the other asset. MARGRABE (1978) suggests to view a cash tender offer as an exchange option. Both bidder and target hold the exchange option value, while target is giving up this value when a cash offer has been launched. Therefore, the

target should charge the bidder a premium to compensate the loss of exchange option value. He concludes that the value of such an option depends not only on the current values of the assets that might be exchanged, but also on the relationship between the rates of return on the two assets, and the duration life of the option.

PINDYCK (1988) examines capacity choice and expansion, based on the assumption that firms cannot continuously and incrementally add capital. PINDYCK (1988) argues that a firm's capacity choice is optimal when the balance between the present value of expected cash flows from a marginal unit of capacity and the total costs of that unit is achieved. Therefore he suggests to calculate the value of an extra unit of capacity first and then move on to determining the value of the option to invest in this unit, together with the optimal timing for exercising the option.

BRENNAN AND SCHWARTZ (1985) is the first attempt to value investment projects in natural resources using a risk-neutral framework. In their analysis, the spot price of the commodity is assumed to follow a geometric Brownian motion, which is a standard process to model stock prices in the option pricing literature.

Most of the earlier literatures, despite their tremendous theoretical contribution, are not able to explain some real-life projects that are more complex in that they involve more than one real options. Most importantly, the value of a collection of multiple real options is not the simple sum of individual real option as values of them may interact. TRIGEORGIS (1993) is one of the attempts to explain explicitly the nature of real options interaction. TRIGEORGIS (1993) points out that there are some situations that option interactions are small and therefore simple option value additivity can be a good approximation. However, the situations that option interactions have a significant impact on the combined value of a collection of real options are also identified and therefore invalidate option additivity. Besides, the option interactions effect can be either positive or negative and TRIGEORGIS (1993) also identifies the conditions for each case.

#### 1.2.4 Strategy and competition

The real options models that have been reviewed so far assume that the firm has a monopoly right to invest in a given project, ignoring the possibility that other firms might enter the market and compete for the same project. As a result, a firm that owns the option to invest or disinvest a project can exercise its real options in a totally free manner. In other words, the value of the real

options owned by the firm is unaffected by any strategic consideration arising from potential competition.

Unfortunately, most firms do not have monopoly right to invest in practice. They may have to consider the possible entry of new competitors, or further expansion of existing competitors. The value of option to wait is inevitably impacted by the feature of competition. For simplicity, consider a industry with a large number of firms and each firm acts competitively. Each firm has the capacity to produce one unit of output, activated by incurring a fixed cost. The price of output is essentially affected by two aspects: the current output flow of the whole industry and a factor that represents uncertainty (which can be of firm-specific, or industry, or both). Non-strategic real options theory would suggest the investment is justified only when the current output price exceeds the sunk cost by a large margin, the value of which is given by the option value of waiting. However, firms cannot wait until its “optimal” timing in a non-competition world as the fear of competition would make it invest early. The value of option of waiting is cut by the nature of competition, and in some extreme cases even drops to zero. The output price cannot be set exogenously as it is in a monopoly framework. It must be treated as an endogenous variable of industry equilibrium. The above example is only one of the possible forms of competition and different competition forms would result in different impacts on the real options value. Therefore, it raises a fundamental doubt concerning the earlier literature and scholars start to appreciate the importance of incorporating competition effects.

This subsection focuses on option game models and also covers several papers discussing strategy. Option game models are the combination of option pricing theory and game theoretic concepts. Option pricing and game theory are complementary theories. The first one deals with maximization of value but ignore the strategic interaction among competitors. Game theory is designed to deal with this caveat. On the other hand, real options approach fills the gap where game theory pays very little attention to payoffs in details and lacks links to market values.

SMIT AND ANKUM (1993) is one of the earliest attempts to combine a real options analysis and a game-theoretical approach. Using a simple binomial model, they conceptually illustrate the impact of competition on project value and its optimal decision rules. They conclude with suggesting different investment tactics to firms with various market positions in terms of their competitive advantage.

GRENADIER (1996) develops a leader-follower game-theoretical real options model in the real estate market. The value of investing options owned respectively by leader and follower is obtained and the equilibrium exercise strategies

are achieved consequently. GRENADIER (1996) sheds light on the real options game research in a duopoly setting.

LUEHRMAN (1998) provides a conceptual framework to explore how option pricing can be applied in improving decision making in terms of the timing and the sequence of a portfolio of strategic investments. In addition to the choice of developing multiple projects in sequence, CHILDS ET AL. (1998) argue that they can also be developed in parallel. They develop a real options model to provide a decision mechanism and consequently determine which investment policy is optimal. They find that the variances of the projects, the correlation coefficient between projects' present values, the development and implementation costs and the time duration for development all have significant impacts on the decision. They conclude that a high correlation, a low volatility, and a high implement cost will favor the choice of sequential development in contrast with parallel development. Besides, their results lead to an interesting argument that it is not always optimal to develop the project with higher net present value. Volatility plays a very important role as the development of high variance project is associated with significant learning.

SMIT AND TRIGEORGIS (1993) argue that some early investment commitments such as R&D investment or an initial plan in a new market, should be treated strategically as a first necessary link in a chain of interrelated investment decisions. With this consideration, the initial investment, which may seem unfavorable if considered in isolation, still provides considerable strategic value and therefore can be optimal. According to their analysis, the decision to implement such a pioneer investment faces a tradeoff between two effects: the flexibility effect and the net commitment effect. The first effect arises from a standard option value of waiting for better information. In short, it is the benefit from flexibility. The net commitment effect indicates that early strategic investment, which would sacrifice flexibility effect, may lead to an indirect impact on the competitor's reaction and the resulted competitive equilibrium. In some extreme cases, the entire market structure will alter and the subsequent optimal investment strategy will be applied accordingly. To summarize, the net commitment effect is the effect to improve the firm's strategic position and enhance the value of its future growth opportunities.

KULATILAKA AND PEROTTI (2001) conduct an analysis on strategic growth options. In their model, the initial investment is interpreted as the acquisition of growth opportunities relative to competitors, which will be exercised if market conditions are favorable. In line with other game-theoretical real options models, they argue that both strategic value, arising from a preemptive effect to improve

the strategic competition position, and flexibility value, arising from the alternative value of not investing, have to be taken into account. Through an imperfect competition model, they show that uncertainty (risk) may not always encourage the choice of not investing, which is contrary to the standard results obtained in the real options literature. Uncertainty may favor the investment when extra profits from strong market share because a better strategic competition position dominates its downside risks.

GRENADIER AND WEISS (1997) form an analogy between the adoption of innovations and the exercise strategy of a chain of future real investment options. For example, to upgrade to a new generation of technology can be treated as holding an option to exchange one innovation for the next one. Therefore, the investment in the early stage of a new technology not only includes the value of current innovation itself, but also incorporates the future option to upgrade. This kind of feature can be discussed within a compound option framework. They find that a firm's optimal migration strategies are path-dependent, which means that two firms facing the same choice will still choose differently due to the fact that they have made different previous investment decisions.

GRENADIER (1999) models the information through the exercise of real options in an imperfect information setting. GRENADIER (1999) points out that each agent in the market holds some private information, which is conveyed through their respective exercise strategy. Each agent exercises its own investment options not only depending on its own signal, but also on the revealed signals of other agents. Essentially each agent faces a tradeoff between early exercise to receive greater potential payoff and the benefits of waiting for information revealed by other market participants' actions. GRENADIER (1999) concludes that consequent market equilibrium exercise will be sequential, with the agents who own the most information exercise first and therefore reveal information to the least informed agents, or does not exercise.

HUISMAN AND KORT (1999) combine the framework of FUDENBERG AND TIROLE (1985) with a real options analysis. They consider two identical firms which both have the possibility to make an investment that generates some uncertain payoff. As two firms are operating in the same market, and the investment decision of one firm will therefore have an impact on its rival's payoff and its exercise strategy. Their model identifies three scenarios. In the first scenario that a preemption equilibrium occurs, firms invest in a leader-follower manner. In the second scenario, two firms invest simultaneously when demand is large enough. The third scenario leads to two possible outcomes: a preemption equilibrium when uncertainty is low and a collusive equilibrium (both firms invest in the



same time) when demand is large. They furthermore argue that the leader in the preemption equilibrium has to sacrifice some of its waiting option to preempt its rival and therefore has a lower threshold than in a monopoly situation. It shows explicitly that competition erodes the option value to wait. Their results also show that, compared with a monopoly case, the strategic option value of waiting is essentially the same in a collusive equilibrium.

WEEDS (2002) also adopts the continuous time framework of FUDENBERG AND TIROLE (1985) and examines a winner-takes-all patent system. There are several forms of uncertainty involved: the uncertain technological success of the project and stochastic economic value of the patent to be won. Again, in a two-player game, each player has to take into account both the value of waiting and the fear of preemption. WEEDS (2002) discovers similar results with HUISMAN AND KORT (1999) in a preemptive leader-follower non-cooperative equilibrium. He also points out that the non-cooperation will delay investment as each holds back from investing in the fear of starting a patent race.

GRENADIER (2002) provides a general solution for equilibrium investment strategies in a Cournot-Nash oligopolistic framework. He finds that the Nash equilibrium exercise strategies are identical to those obtained in an “artificial” perfectly competitive equilibrium, with a modified demand function. Therefore what seems a more complex case of oligopolistic settings can be solved from the results of models where there is perfect competition.

In a more recent work, LAMBRECHT AND PERRAUDIN (2003) consider entry exercise strategies with preemption effects under incomplete information. They find that the value of waiting delays investment while the fear of preemption speeds up the investment and therefore the investment threshold in a multiple firm equilibrium lies somewhere between the zero-NPV threshold and the optimal strategy of a monopolist. The level of the threshold depends on the distribution of competitors’ costs and the implied fear of preemption.

They consider a firm  $i$ , with an investment opportunity that generates an output price, or a flow of profit  $P$  by incurring a sunk cost  $X_i$ .  $P$ , without loss of any generality, follows a standard geometric Brownian motion:

$$(1.71) \quad dP = \mu P dt + \sigma P dz.$$

The above equation indicates that the output price, as well as the profit flow generated from the project, is expected to grow at the trend rate  $\mu$ . I start from analyzing the value of investment opportunity in a monopolistic setting, denoted by  $V$ . It is contingent on the value of the basic asset  $P$ , which enables us to price  $V$  with the procedure of contingent claims valuation.

A riskless portfolio consisting of the asset to be valued ( $V$ ) and basic asset ( $P$ ) is constructed. As the portfolio is riskless, it must earn the risk-free rate of return. The standard procedure yields a differential equation for the unknown value of the project which can be solved subject to boundary conditions.

Consider a portfolio at time  $t$ , which contains one unit of the project value,  $V(P)$ , and a short position of  $n$  units of output price,  $P$ . During an infinite small interval of time  $(t, t + dt)$ , the value of  $n$  can be seen as a constant. The value of  $n$  should also make the portfolio riskless, and therefore should equal to  $V'(P)$ .

The holder of the project will receive revenue  $Pdt$  over the time interval of length  $dt$ . The short position in this portfolio must require a payment that equals to the dividend yield, namely,  $\delta Pdt = (r - \mu)Pdt$ ; otherwise no rational investors will enter into the long side of the transaction. Here  $r$  represents a proper discount rate, the value of which can be obtained through capital asset pricing model.

Consequently, holding the portfolio yields net revenue  $(P - n\delta P)dt$ . On the other hand, the capital gain of the portfolio should have the value  $dV - V'(P)dP$ . Therefore total return ( $\Phi$ ) from holding the portfolio over a short time interval  $dt$  is:

$$(1.72) \quad \Phi = dV - V'(P)dP + Pdt - \delta PV'(P)dt.$$

Using Ito's Lemma to expand  $dV$  and rearranging yields:

$$(1.73) \quad \Phi = [P - \delta PV'(P) + \frac{1}{2}\sigma^2 P^2 V''(P)]dt.$$

The return  $\Phi$  must equal the riskless return  $r_f[V(P) - V'(P)P]dt$ . Thus,  $v(P)$  must satisfy the following differential equation

$$(1.74) \quad \frac{1}{2}\sigma^2 P^2 V''(P) + (r_f - \delta)PV'(P) - r_f V(P) + P = 0.$$

A general solution to the above equation is a linear combination of two independent solutions  $A_1 P^{\beta_1}$  and  $A_2 P^{\beta_2}$  and any particular solution of the equation such as  $P/\delta$ .  $\beta_1$  and  $\beta_2$  are the two roots of the fundamental quadratic equation

$$(1.75) \quad Q \equiv \frac{1}{2}\sigma^2 \beta(\beta - 1) + (r_f - \delta)\beta - r_f = 0.$$

It is also worth mentioning that  $\beta_1 > 1$  and  $\beta_2 < 0$ .

Thus, the solution of  $V(P)$  can be expressed by

$$(1.76) \quad V(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2} + P/\delta,$$

where the  $A_1$  and  $A_2$  are yet to be determined by the boundary conditions.

According to DIXIT AND PINDYCK (1994), the value of the project contains two parts: the fundamental component ( $P/\delta$ ) and the speculative components ( $A_1P^{\beta_1} + A_2P^{\beta_2}$ ).

The fundamental component of the value of the project can be interpreted as the expected present value of the revenue stream  $P_t$  when the initial level is  $P$ . It can be proved by the following analysis. Considering the features of geometric Brownian motion, it is given that  $\mathcal{E}[P_t] = Pe^{\mu t}$ . Discounting at an appropriate discount rate  $r$  gives:

$$(1.77) \quad \int_0^\infty Pe^{\mu t} e^{-rt} dt = P/(r - \mu) = P/\delta.$$

However, the speculative component,  $A_1P^{\beta_1} + A_2P^{\beta_2}$ , should be ruled out.

First, it makes sense that  $V(0) = 0$ . According to the features of the geometric Brownian motion, if the price is ever zero, it will forever stay at zero and therefore the value of the project should also stay at zero. The above analysis provides that  $A_2$  must be zero.

The other term,  $A_1P^{\beta_1}$ , interpreted by DIXIT AND PINDYCK (1994), represents a speculative opportunity that “people might value the asset above its fundamentals if they expected to be able to resell it later at a sufficient capital gain”<sup>2</sup>. Such speculative opportunity must be ruled out and then the value of project should only contain its fundamental component:

$$(1.78) \quad V(P) = P/\delta,$$

Therefore, the value of the project,  $V$ , being a constant multiple of  $P$ , should also follow a geometric Brownian motion with the same drift and diffusion parameters. Hence the investment problem can be reduced to the model aforementioned.

Let  $F(P)$  denote the value of investment project as a function of the output price. It must satisfy the following differential equation

$$(1.79) \quad \frac{1}{2}\sigma^2 P^2 F''(P) + (r_f - \delta)PF'(P) - r_f F(P) = 0,$$

via a contingent claims analysis;

or

$$(1.80) \quad \frac{1}{2}\sigma^2 P^2 F''(P) + \mu PF'(P) - rF(P) = 0,$$

through a dynamic programming analysis.

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<sup>2</sup> See DIXIT AND PINDYCK (1994) for further detail.

Again, the above two equations are identical under a risk neutral assumption, i.e.,  $r = r_f$ .

The above differential equations can be solved by the following three boundary conditions:

$$(1.81) \quad F(0) = 0,$$

$$(1.82) \quad F(P_i^*) = V(P_i^*) = P_i^*/\delta - X_i,$$

$$(1.83) \quad F'(P_i^*) = V'(P_i^*) = 1/\delta.$$

Thus, the optimal investment rule, i.e., the threshold of output price above which it is optimal to complete the project, can be expressed by

$$(1.84) \quad P_i^* = \frac{\beta_1}{\beta_1 - 1} \delta X_i.$$

Upon exercise, the value of investment project has the value given by

$$(1.85) \quad V_i^* = P_i^*/\delta = \frac{\beta_1}{\beta_1 - 1} X_i.$$

Therefore, the value of the firm prior to investment is

$$(1.86) \quad V = \left( \frac{P_i^*}{\delta} - X_i \right) \left( \frac{P}{P_i^*} \right)^{\beta_1}.$$

If the firm is threatened by preemption, intuitively the value of option to wait will be reduced. For simplicity, consider there exists only one competitor, firm  $j$ , which is an otherwise identical firm with an investment cost  $X_j$ . As equation (1.85) indicates, a different cost will result in a different investment threshold. If the threshold  $P_j^*$  is lower than  $P_i^*$ , firm  $i$  will be preempted by firm  $j$  and loses any further opportunity to invest. Therefore, firm  $i$  must determine its threshold by considering the threat of preemption, i.e., the probability that it has the lower threshold.

To introduce incomplete information, they assume that firm  $i$  conjectures that firm  $j$  invests when  $P$  first hits its threshold  $P_j^*$ . However firm  $i$  only knows the distribution of  $P_j^*$ , denoted by  $F_j$ .  $F_j$  has a continuously differentiable density function  $F_j'$  with positive support on an interval,  $[P_L^*, P_U^*]$ .

Thus the probability that firm  $i$  can preempt its competitor or firm  $i$  has the lower threshold is described by  $1 - F_j(P_i^*)$ , which depends on its own investment threshold. Therefore the value-matching condition and smooth-pasting condition must be adjusted accordingly:

$$(1.87) \quad V_i(P_i^*) = [1 - F_j(P_i^*)][(P_i^*)/\delta - X_i]$$

$$(1.88) \quad V_i'(P_i^*) = [1 - F_j'(P_i^*)][(P_i^*)/\delta - X_i] + F_j(P_i^*)/\delta$$

Solving them yields the equation that the threshold  $P_i^*$  must satisfy:

$$(1.89) \quad P_i^* = \frac{\beta_1 + \frac{P_i^* F_j'(P_i^*)}{1 - F_j(P_i^*)}}{\beta_1 - 1 + \frac{P_i^* F_j'(P_i^*)}{1 - F_j(P_i^*)}} \delta X_i.$$

The above results provide some interesting insight to the effect of competition. Let  $P_{NPV}^*$  denote the investment threshold suggested by NPV rules. It is easily obtained that  $P_{NPV}^* = \delta X_i$ . To avoid confusion, notions are re-organized. The optimal threshold for firm  $i$  facing no competition is  $P_{Monopoly}^* = \frac{\beta_1}{\beta_1 - 1} \delta X_i$  and the one that firm faces the threat of preemption is  $P_{Preemption}^*$ , the value of which is given by the above equation.

To compare the these thresholds, one must evaluate the value of the term  $\frac{P_i^* F_j'(P_i^*)}{1 - F_j(P_i^*)}$ , which clearly is non-negative. Therefore the following relationship must hold:

$$(1.90) \quad P_{NPV}^* \leq P_{Preemption}^* \leq P_{Monopoly}^*.$$

It shows that the optimal investment strategy when incomplete information and preemption are introduced may lie anywhere between the zero-NPV threshold and the optimal strategy of a monopolist, depending on the conjecture of the rival's threshold.

In fact, the threshold obtained through the model converges to either case. If the fear of preemption is very small, in an extreme case, the chance of winning for firm  $i$  ( $1 - F_j(P_i^*)$ ), will simply be one. Even if it increases its threshold, it does not affect the probability of winning. Therefore, the derivative,  $F_j'(P_i^*)$ , will be zero. It makes the threshold converge to  $P_{Monopoly}^*$ . On the other hand, if the fear of preemption is sufficiently large, firm  $i$  believes there is no chance that its threshold will be lower than that of its rival. The term  $1 - F_j(P_i^*)$  will have the value of zero.

In a later work, SHACKLETON ET AL. (2004) consider optimal entry strategies in a two-firm, infinite-horizon stochastic game. They assume that market capacity is only big enough for one firm but its idle rival has the option to reclaim the market whenever optimal to do so. Their paper makes the first attempt to solve a two-state-variable strategic game as the model allows for different but correlated stochastic processes associated with each firm's net operating profitability. They point out that if entry costs are ignored, the firm with the highest opportunity cost, i.e., the firm who would realize the highest current payoff by exercising its entry option, will operate in the market. With the existence of fixed costs to enter the market, the hysteresis effect is found in the entry decisions of competing firm. The effect of hysteresis is positively related to investment entry cost and volatility, but negatively related to the correlation of rival firms operations.

MASON AND WEEDS (2005) find similar results with KULATILAKA AND PEROTTI (2001) that a higher volatility may provide an incentive to hasten rather than delay investment, under a duopoly competition model. It is mainly due to the impact of uncertainty on the equilibrium outcome of the timing game among players.

In a very recent work, PAWLINA AND KORT (2006) examine the situation where two firms have opportunities to invest in a profit-enhancing investment project but need to face different investment costs. Three different equilibrium strategies are obtained. First, a relatively small asymmetry in investment costs will result in a relatively small first-mover advantage and therefore two firms are optimal to invest at the same time. Second, in the case where the first-mover advantage is large enough, the firm who owns the costs advantage will invest first and preempt its rival. Third, in the situation where asymmetry both in costs and first-mover advantage are sufficient, a sequential investment action will be observed.

Real options game models are well summarized by DIXIT AND PINDYCK (1994) (Chapter 8 and 9), TRIGEORGIS (1996) (Chapter 9) and SMIT AND TRIGEORGIS (2004).

### 1.3 M&A and real options

Mergers and acquisitions are one of the most well known investment approaches for corporate managers. They share the exact same three characteristics with an investment opportunity in general as I have discussed before. First, the takeover process is partially or completely irreversible. Once the deal is processed and the payment is made, there is no turning back. Second, the value of the target firm involves a great deal of uncertainty, which may be impacted by macroeconomic conditions, politic conditions or even market sentiment. The value of takeover payment, if made in shares of bidder firm, also fluctuates over time. Third, corporate managers are able to choose the timing to conduct the deal, with at least a certain amount of flexibility. Managers have the right to make their takeover decisions when market favors the deal the most.

As a consequence, mergers and acquisitions have become one of the subjects investigated widely by a real options approach. SMITH AND TRIANTIS (2001) make one of the early contributions. They show that through a series of acquisitions over time, long-term acquisition programs can significantly alter a bidder's competitive position (and even the structure of its industry) with the development of growth options. Takeovers also involve flexibility options and divestiture options. Flexibility options arise when firms with significant flexibility in

organization, marketing, manufacturing and financing may reap additional benefits from strategic acquisitions involving diversification and acquiring flexible resources. Divestiture option value is created by the option to divest parts/all of the acquired companies at a later date, and future sale of these assets may substantially limit downside risk. Real options make a considerable contribution to the shareholders value in their framework.

Another conceptual paper regarding takeovers options is contributed by DAPENA AND FIDALGO (2003). They model tender offers as a combination of a waiting option and a growth option. Waiting option value is created through sequential investment instead of investing at once, through purchasing minority shareholdings, investors own potential growth options that carry all private benefits they can seize by making follow up investments. From their perspective, information releasing is crucial in deciding the exercise price of a growth option.

SMIT (2001) points out that some initial transactions, which are part of a “buy-and-build” acquisitions strategy (the initial acquisition plays a strategic role for the follow-on transactions to provide a platform that later deals can build on and leverage core efficiencies onto), cannot be treated properly by a traditional NPV analysis. The strategic value that traditional NPV analysis ignores can be sufficiently large and have a significant impact on the decision mechanism for such acquisition deals. Similarly, the competitive feature of acquisitions bidding contest cannot be ignored either. SMIT (2001) also discusses the strategic interaction among competing bidders and shows possible strategies of each bidder, considering their respectively market position. Essentially, he suggests treating the strategic acquisition as a package of corporate real options actively managed by the firm in a context of competitive responses or changing market conditions. In his model, standard acquisition with no follow-on actions is the option to “exchange” the target value to the buyer against a future price and platform acquisition is viewed as the compound options in a series of synergistic deals.

Early research on real options approach to mergers and acquisitions sheds light on the concepts of various options captured by a corporate takeover deal. Under these frameworks, scholars have made significant contributions to a more rigorous and technical analysis in continuous time setting.

LAMBRECHT (2001) examines three different forms of M&A activities including mergers, stock offers and cash offers in a real options framework. He models the firms’ instantaneous profit as the Cobb-Douglas production function with decreasing returns to scale, therefore the firms’ profit is a convex function of its output prices. This has important consequences with respect to the effect of uncertainty on takeover decisions and the model’s feasibility to account for

economies of scale. In his model, mergers are modelled as a Nash equilibrium in which both companies simultaneously determine the timing and terms of the deal. Because of this feature, the timing of mergers is efficient from a global optimization prospective, i.e., it happens at the time when the value of synergy is maximized. Stackelberg leader-follower game is employed to model the stock and cash offers. In these offers, essentially the target first determines the terms of the deal, and the bidder subsequently determines the time. The separation of timing and terms decision results in inefficiency and therefore stock offers and cash offers will happen inefficiently late compared with mergers or the threshold obtained from a global optimization approach. Besides, he finds that cash offers are even more inefficient because of its non-participation effect as the exchange of cash with its company shares prevents target firm from having a stake in the newly created firm. Consequently, the payoff structure and the incentives of the bidder and the target are not aligned. In other words, the bidder has the incentive to delay the transaction and wait for a lower bidding premium, while the target does not have such incentive because it does not hold any real options. In terms of the returns, bidders get the highest returns in cash offers and targets prefer stock offers. LAMBRECHT (2001) provides theoretical support to the empirical findings that merger and acquisitions activities are cyclical. LAMBRECHT (2004) develops his previous work by incorporating costs in the production function and has a brief discussion on the impact of market power.

BERNILE ET AL. (2006) make another attempt to answer why mergers happen in the pattern of waves. They consider a horizontal merger that is motivated only by strategic considerations. It is demonstrated through a real options analysis that firms' propensity to merge is highest during periods of extremely high and extremely low demand, which provides a theoretical explanation to merger waves. The results are generated from strategic interaction among the incumbents and the potential entrant. The decision to merge another firm in the industry is affected by a trade-off between enhancing the incumbents' combined profits and the possibility of new entry to erode the incumbents' profits, which is triggered by higher prices and higher potential outsiders' profits. In the case of high demand, the incumbents cannot prevent entry and therefore tend to conduct a merger deal anyway to increase profits. During periods when the demand is very low, even if the merger deal is executed, the market condition is still too bad to trigger any new entry. Consequently, the incumbents will be better off merging. To summarize, the incumbents' merging decision has a very limited impact on the entry decision, and the value enhancement considerations dominate the decision to merge.

LAMBRECHT (2004) abstracts from two important issues in takeover deals: com-



petition and imperfect information. He assumes that bidder firm has the monopolistic right to enter into a takeover deal and in his model the deal participants have complete information. MORELLEC AND ZHDANOV (2005) study the takeover game in the context of competition and imperfect information. They model the competition in which two potential bidders differ in terms of synergy benefit and transaction costs. The critical difference is that target is put into an advantageous position and as a result the bargaining power of bidder is decayed. Takeover contest is modelled as an ascending English auction in which the bidder with the highest bid wins the competition. Imperfect information from outside investors rather than efficient market leads to abnormal stock returns on the announcement of the takeover. In addition, MORELLEC AND ZHDANOV (2005) rely on two different degrees of correlation between the bidder and the target stock returns, while LAMBRECHT (2004) considers only one source of uncertainty that mostly applies to horizontal mergers. Through exchange option analysis, MORELLEC AND ZHDANOV (2005) predict that returns to target from takeover are asymmetrically higher than returns to bidder, and returns to bidder could be negative in some extremely competitive cases. Their findings are consistent with empirical evidence.

SMIT ET AL. (2005) present another takeover competition real options model with imperfect information. However, their study differs in terms of the nature of imperfect information compared with MORELLEC AND ZHDANOV (2005). MORELLEC AND ZHDANOV (2005) assume that outside investors have incomplete information and can update their beliefs by observing the behavior of participating firms. Nevertheless, in SMIT ET AL. (2005), there exists information asymmetry even among the participants and information is not free. Potential uninformed bidders need to incur due diligence costs (costs to purchase the takeover options) to acquire the information regarding their respective target value expectation (underlying value of the takeover options) before making a bid (exercise price). Implementing asymmetric and costly information auction to takeover bidding process, the model presents a two player setting, where the initial bidder may decide to make a preemptive or an accommodating bid after conducting due diligence. When the initial bidder makes a preemptive bid, the cost to buy the option is higher than the second bidder's takeover option value. Therefore, the second bidder chooses not to enter the bidding contest and the first bidder acquires the target at this preemptive bid. On the other hand, when the initial bidder accommodates the competition, an English auction opens up and the bidder with the highest valuation of target wins the auction, with the right to acquire the target at the second highest bid. Their model leads to a number of predictions: for example, more heterogeneity (lower correlation between

potential bidders) generates higher value appropriation. However, very high level of correlation also leads to a rise in value appropriation under incomplete information.

The models reviewed so far consider takeover deal as a deterring option. They investigate into the optimal timing when bidder makes its bid towards the target, with or without competition threat. However, when market is unfavorable, corporate management may want to divest the firm purchased earlier and realize resale value through divestment. LAMBRECHT AND MYERS (2007) develop a real options model of takeover disinvestment when the market is declining. They consider a public firm with dispersed outside shareholders. They consider an agency problem that managers do not act to maximize outside shareholder's value but to focus on their own welfare. However, agency problem is controlled by the fear of managers that outside shareholders are able to exercise their property rights and take control of the firm. They furthermore assume that there is a cost associated with collective action, otherwise the agency problem can be easily prevented. In order to avoid that, managers need to provide enough money to pay to the outside shareholders. Their results show that agency problems do damage the value: managers always tend to delay the abandonment when demand declines and therefore miss the optimal timing when the shareholder value is maximized. As a device to solve this agency inefficiency, golden parachutes are introduced. However, LAMBRECHT AND MYERS (2007) find that golden parachutes can only alleviate the inefficient delay rather than eliminate it. An "optimal" golden parachute that would induce the manager to choose the closure time in the first best closure is always too expensive for outside shareholders and therefore cannot obtain their approval. They also consider the impact of financial leverage on the managers' decision and discover an optimal debt level to achieve the most efficient abandonment. In the last part of their analysis, they compare four alternative takeover mechanisms and their respective impact on the efficient abandonment. Financial investors take over the firm at exactly the right level of product demand and therefore pursue the first-best outcome. A hostile takeover might involve a takeover contest and therefore the fear of preemption might push the bidder to invest earlier than the first-best timing. Management buyouts (MBOs) and mergers of equals are found to lead to inefficient delay as either managers lose the ability to capture cash flow when they take over and shut down or negotiation between two firms' managers reduces the power of the target shareholders to extract value from the bidder.

MORELLEC AND ZHDANOV (2006) also analyzes the interaction between financing strategies and takeover activities. Their model assumes that both bidder and target firms are levered. Their results indicate that in an asymmetric equilibrium

of financial leverage, the bidder with the lowest leverage wins the takeover battle. The winner of the takeover contest is expected to lever up after the takeover deal. In equilibrium, the leverage ratio of the winning bidder is substantially lower than the leverage ratio that is optimal to balance tax benefits of debt and bankruptcy costs, and it helps explain why two otherwise identical firms choose the different financing strategies.

Most recently, empirical evidence on the explanatory power of real options starts to emerge. MORAN AND BETTON (2004) model acquisition as a Stackelberg leader-follower game with complete information using real options techniques. They test the hypotheses related to acquisition premium with a sample of 228 US target firms during the period of 1982-2001. They find that target volatility and market to book ratios are important determinants of acquisition premiums, which is consistent with the model predications.

DUNIS AND KLEIN (2005) choose a small sample covering 15 mergers and acquisitions in the European financial services industry in late 1990's. They discover that from an option theory point of view, those deals are not overpaid as the option premium almost doubles the actually paid takeover premium. Besides, their results indicate that the actual volatility is much higher than the implicitly assumed volatility and post-merger performance is overestimated. However, the sample size of 15 significantly limits the implication of work by DUNIS AND KLEIN (2005). CAMPBELL AND KRAUSSL (2006) extend their sample to 100 mergers and acquisitions in the European banking industry. They find that average takeover option premium is 14% and both the size of the bidding banks and the debt to equity ratio of target banks significantly affect this premium.

Following the model developed by MORELLEC AND ZHDANOV (2005), HACKBARTH AND MORELLEC (2006) furthermore discuss the impact of a takeover deal on the stock returns of participating firms and provide some empirical evidence to support their propositions. The timing and terms of takeovers are determined by a Nash equilibrium that two firms determine the timing and terms of deal simultaneously. The model indicates that the beta of the bidder will achieve an appreciation prior to the announcement, and a depreciation at the time of the announcement, when the firm has a higher pre-announcement beta than the target firm. On the other hand, if the bidding firm has a lower pre-deal beta than its target, its beta will experience a drop before the announcement and a rise at the time of announcement. The empirical evidence obtained from a sample of 1090 takeovers of publicly traded US firms in the period between 1985 and 2002 is consistent with the model predictions to some extent.

## 1.4 Research questions

Academic research dealing with the application of option pricing theory to valuing real assets have appeared in the finance literature since MYERS (1987). However, practical application of these ideas, especially in mergers and acquisitions industry, takes place mainly in the last several years.

Most of the current real options takeover models fall into one of the following two categories: the valuation of a takeover as an individual investment project, and takeover related financing and investment decisions. The papers in the first category discuss several most important questions involved in the valuation of a takeover deal: how much should a bidder pay, how much should a target require, when is optimal to make an offer, how is the competition going to affect decision making on both sides, and how does the information content result in asymmetry, etc. (Such as LAMBRECHT (2004) and MORELLEC AND ZHDANOV (2005)) The second category, however, focuses on the possible impacts of taxes, asset liquidity, debt policy and manager-shareholder conflicts on takeover decisions. (Such as LAMBRECHT AND MYERS (2007) and MORELLEC AND ZHDANOV (2006))

My research belongs to the first category mentioned above. It aims to develop a dynamic real options model for the joint determination of price and decision rules of takeovers under uncertainty. It, however, has its focus on one of the most important aspects in takeover transactions: the payment choice of the deal. I aim to fill, to the best of my knowledge, a research gap where current literature lacks of discussion. A choice of takeover payment method, such as cash, shares or a mixture of cash and shares, certainly has an impact on the valuation of the deal and the decision making strategy of both participants. In terms of a simple real options analysis, a pure cash offer represents a deal exchanging a stochastic asset with a non-stochastic asset. A pure share offer, on the other hand, involves exchanging two stochastic assets. It is well known that in real options analysis, the degree of uncertainty decision-makers face greatly influences their decisions. Everything else being equal, a disparity in uncertainty undoubtedly will lead to different strategies. This is why I believe it is curial to investigate the properties of the payment form threshold in a takeover deal.

The thesis is designed to answer the following questions: 1. What is the payment form threshold in a takeover transaction? 2. Which factor impacts the payment form threshold and to what extent? 3. If a deal is made by the payment form threshold, how is the synergy allocated among participants? Empirical evidence is also provided for the model predictions at the end of the thesis.

## Chapter 2

# Takeovers threshold for a cash offer takeover

This chapter introduces a basic model providing a fundamental framework for all the models discussed throughout the thesis. Most of the model assumptions that are laid out below will apply to the subsequent models to a certain degree. The model relies on the analogy between takeover opportunities and deferring options for both bidder and target firms and therefore the decision to conduct a takeover is modelled as a strategic investment decision under uncertainty, which calls for the need to conduct a real options (in continuous time) analysis.

## 2.1 Model assumptions

Consider a situation where one company (the bidder) approaches another company (the target) with an offer to purchase all its outstanding shares against cash at a price  $X$ . For simplicity, the model does not consider partial control of the target firm, i.e., the purchase of only part of target firm's outstanding shares. While the focus will move on to a variety of payment forms, the current chapter looks at the pure cash offer only. It is further assumed that cash is available in any required quantity at no additional costs.

The model looks at the takeover deals from a different way than most of the other research in the literature. For example, MORELLEC AND ZHDANOV (2005) assume both bidder and target shareholders are attempting to maximize the synergy effect (i.e., as an endogenous factor that can be affected by participants' decision) generated from the deal first and then determine the allocation rule of the synergies. In my model, in contrast, the synergy effect is defined as a percentage of the value of the target firm and this percentage value is an exogenous factor that cannot be influenced by the decision of any takeover participants. Each participant's takeover options value give rises to a specific synergy effect threshold required. The takeover deal will not go through unless this threshold

is met (as discussed in the following chapter, different payment forms can lead to various levels of thresholds, which therefore gives role to a threshold payment form that requires the lowest level of synergy effect threshold).

The model set-ups allow for the role of a synergy effect threshold that links the incentive of two takeover participants. It will be shown in the subsequent analysis that a deal will only be observed if the synergy effect is not less than a certain threshold. The rationale is simple. Both bidder and target require a premium to compensate their respective real option and a deal will only emerge when the synergy effect is large enough to accommodate both option premiums in the same time.

Bidder and target are acting to maximize the values for their respective shareholders, thus ignoring any agency problems. All the takeover costs such as the fees paid to investment bankers and lawyers are assumed to be insignificant to affect the takeover decision and then are ignored. There exists a constant discount rate  $r$ . Both bidder and target are risk neutral and therefore the discount rate  $r$  is just the risk-free rate. The model ignores the “free-rider” issue by assuming one individual shareholder controlling the whole firm.

The source of uncertainty relates to the cash flows from the businesses of the target firm. In particular, it is assumed that the present value of these cash flows (denoted by  $S_T$ ) is observable and governed by the following stochastic differential equations:

$$(2.1) \quad dS_T = \mu_T S_T dt + \sigma_T S_T dz_T,$$

where  $\mu_T$  and  $\sigma_T$  are respectively the drift (growth) and diffusion (volatility) parameters and the target firm has complete information regarding the stochastic property of  $S_T$ . In the same time, the stochastic property of  $S_T$  is also available for the bidding firm.

On the other hand, the bidder understand that with a potential synergy effect, the target is worth more than  $S_T$ . The value enhancement due to this synergy effect is defined as a fraction of the target firm:  $\alpha S_T$ . It is furthermore assumed that target firm has complete information regarding the potential synergy effect however the bidder firm is assumed to have all the bargaining power and therefore is able to make it take-it-or-leave-it offer. Under these assumptions, it is hence rational for the target to make the takeover decision purely based on the real options optimization, regardless of the existence of the synergy effect.

The value of cash payment is denoted by  $X$  and it links the takeover decisions of bidder and target. Bidder firm pays  $X$ , which is its investment cost, to exchange for  $(1 + \alpha)S_T$ , which reflects its payoff. On the other hand, the target gives up

its own company, valued at  $S_T$  by itself, to receive its payoff  $X$ .

## 2.2 Model developments

With both parties having the flexibility whether to make a bid and whether to accept this bid, respectively, it is appropriate to use a real options approach to model this decision-making process. One of the most important implications from a real options analysis is the hysteresis effect, i.e, even if the short-term conditions favor the decision of immediate investment, waiting can still be optimal. In other words, both bidder and target are optimal to participate such a takeover deal until investment return exceeds its corresponding investment cost by a substantial premium, measured by each firm's opportunity cost. In the following, I will illustrate this idea in more detail.

The analysis starts from assuming that both sides employ a traditional NPV method. In a NPV analysis, investors should take a positive NPV project and reject the one with negative NPV. As a consequence, as long as the takeover surplus is marginally larger than 0 for both bidder and target in the same time, one should be able to observe a takeover.

Mathematically, the conditions for a takeover to happen in the NPV framework should be given by <sup>1</sup>

$$(2.2) \quad \text{NPV}_{\text{Bidder}} = (1 + \alpha)S_T - X \geq 0,$$

and

$$(2.3) \quad \text{NPV}_{\text{Target}} = X - S_T \geq 0.$$

Combining these two equations, it can be obtained that as long as the synergy effect  $\alpha$  is marginally greater than 0, a takeover transaction should be observed. In other words, the lowest required synergy effect to justify a takeover transaction should be just 0. Apparently, this synergy effect threshold will be constant at 0 regardless of the method of payment.

However, the above argument will no longer hold in a real options framework. It is well known that the optimal investment rule, as described in a real options framework, is to invest when the asset value exceeds the investment cost by a option premium. In other words, it is optimal to invest when immediate net payoff from the project exceeds the value of option to wait, which depends on the uncertainty or the randomness the investor will have to face.

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<sup>1</sup> For simplicity, I ignore all possible costs that might occur throughout the deal.

As a result, the following conditions must be fulfilled to trigger a deal:

$$(2.4) \quad (1 + \alpha)S_T - X \geq F^B,$$

$$(2.5) \quad X - S_T \geq F^T,$$

where  $F^B$  and  $F^T$  are the values of takeover options owned by the bidder and target respectively. Condition (2.4) specifies what the bidder would like to accept to enter a takeover deal and (2.5) describes the condition for target. If and only if both conditions are fulfilled in the same time, a deal is observed. Noting that option value to wait should be positive, I can then conclude that there exists a synergy effect threshold  $\alpha_{min}$  (the lowest required synergy effect), below which two conditions stated above cannot be fulfilled simultaneously:

$$(2.6) \quad \alpha \geq \alpha_{min} = \frac{F^B + F^T}{S_T} > 0.$$

### 2.2.1 Bidder's takeover decision

The value of bidder and target's takeover options is denoted respectively by  $F^B(S_T)$  and  $F^T(S_T)$ . The takeover decision of the bidder firm is derived first. Following a standard real options argument, bidder only enters a deal when the immediate payoff  $(1 + \alpha)S_T - X$  exceeds the opportunity cost  $F^B(S_T)$ , which reflects the value of an option to delay a deal.

Noting that  $S_T^*$  refers to the target's fundamental value upon the time of the takeover, the result is presented below.

**Lemma 1.** *Define  $S_T^*$  as the fundamental value of target at time of takeover, the value of takeover option for bidder is*

$$(2.7) \quad F^B = [(1 + \alpha)S_T^* - X] \left( \frac{X/S_T}{R^B} \right)^{\beta_B} \quad \text{for } X/S_T > R^B,$$

$$(2.8) \quad = (1 + \alpha)S_T - X \quad \text{for } X/S_T \leq R^B.$$

The takeover threshold for bidder is given by

$$(2.9) \quad R^B = \frac{\beta_B - 1}{\beta_B} (1 + \alpha),$$

where  $\beta_B$  has the value of

$$(2.10) \quad \beta_B = \frac{1}{2} - \frac{\mu_T}{\sigma_T^2} + \sqrt{\left( \frac{\mu_T}{\sigma_T^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma_T^2}} > 1$$

The takeover threshold implies that if the ratio of  $X/S_T$  is equal to or less than  $R^B$ , it is optimal for bidder to enter a takeover deal.



*Proof.* See appendix. □

Through a simple re-arranging, one can have the bidder's threshold represented by  $\frac{X}{(1+\alpha)S_T} = \frac{\beta_B-1}{\beta_B}$ . The interpretation of the result becomes easier with this form given  $\frac{\beta_B-1}{\beta_B}$  should be less than one with  $\beta_B$  being larger than 1. As the deal involves uncertainty, the bidding firm is only happy to make an offer if a positive return is expected (i.e., the value of  $(1+\alpha)S_T$  must be larger than  $X$ ) and the gratitude of this positive return is suggested by the above equation.

### 2.2.2 Target's takeover decision

While the opportunity to buy target firm resembles a call option to the bidder, the opportunity to sell its own firm represents a put option to the target. The value of this option still depends on the stochastic property of  $S_T$ . The option to sell differs from the option to buy in the sense that if value of the asset ( $S_T$ ) to be sold approaches infinity, it will never be optimal to sell it and therefore the options value will become worthless.

**Lemma 2.** *Define  $S_T^*$  as the fundamental value of target at time of takeover, the value of takeover option for target is*

$$(2.11) \quad F^T = [X - S_T^*] \left( \frac{X/S_T}{R^T} \right)^{\beta_T} \quad \text{for } X/S_T < R^T,$$

$$(2.12) \quad = X - S_T \quad \text{for } X/S_T \geq R^T.$$

*The takeover threshold for target is*

$$(2.13) \quad R^T = \frac{\beta_T - 1}{\beta_T},$$

*where  $\beta_T$  is has the value of*

$$(2.14) \quad \beta_T = \frac{1}{2} - \frac{\mu_T}{\sigma_T^2} - \sqrt{\left( \frac{\mu_T}{\sigma_T^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma_T^2}} < 0$$

*The takeover threshold implies that if the ratio of  $X/S_T$  is greater than  $R^T$ , it is optimal for target to accept a takeover offer.*

*Proof.* See appendix. □

The lemma suggests that to fulfill the conditions that allow a target to enter the deal, the ratio of  $X$  to  $S_T$  must not be not less than  $(\beta_T - 1)/\beta_T$ , which has a value larger than 1 given  $\beta_T$  is negative. This implies that only when the immediate payoff from the deal  $X - S_T$  exceeds a certain premium target firm is willing to sell its firm.

### 2.2.3 The threshold for the deal

Lemma 1 and 2 essentially give the threshold of each takeover participants. For the sake of tractability, the results are repeated here.

The bidder is willing to make an offer only if

$$(2.15) \quad \frac{X}{(1 + \alpha)S_T} \leq \frac{\beta_B - 1}{\beta_B}$$

The target is willing to accept an offer only if

$$(2.16) \quad \frac{X}{S_T} \geq \frac{\beta_T - 1}{\beta_T}$$

Combining two conditions yields the following lemma

**Lemma 3.** *The takeover threshold that fulfills both the requirements of bidder and target is given by*

$$(2.17) \quad \alpha \geq \alpha_{min} = \frac{\beta_T - 1}{\beta_T} \frac{\beta_B}{\beta_B - 1} - 1,$$

where  $\beta_{B,T}$  are aforementioned. Noting that  $\beta_T < 0$  and  $\beta_B > 1$ ,  $\alpha_{min}$  must be positive.

Please note that bidder firm's stochastic property does not affect either firm's takeover threshold. It therefore has no impacts on the threshold for the deal. In a takeover deal where bidder pays cash to acquire the target firm, bidder firm's business should not have any influence on whether the deal can go through. From the target point of view, the decision is based on the value of the cash payment and what the target firm is worth, which includes its fundamental value and the option to sell the firm. From the bidder's perspective, the decision to acquire the target firm by cash (and as model assumes, the cash is already available) is separate from how its own business performs. Regardless of the value of the bidder firm's fundamental value (which will be introduced in the latter chapter as  $S_B$ ), the bidder firm will only make the offer if the net payoff from purchasing the target firm exceeds the takeover option value.

## 2.3 Conclusions

Using a continuous-time real options approach I derive conditions for a synergy effect threshold under which a takeover can be observed. The threshold to trigger a takeover deal is subject to a variety of variables such as the expected growth rate and volatility parameter of target firm as well as the discount rate. The chapter provides a foundation for further analysis.

## Chapter 3

# The payment form threshold in tender offers

This chapter develops a real options model of takeovers in which the bidder makes a direct tender offer to purchase all the shares of the target firm and both participants have a choice out of three possible payment methods: pure cash payment, pure share payment or a combination of cash and share. The takeover participants resemble their respective takeover opportunity as an deferred option, which defines the takeover threshold for both sides. Takeover options, however, have different values associated with different methods of payment. As a result, the takeover threshold differs in various payment forms. In each case, a synergy effect threshold that is able to trigger a deal is derived and compared analytically. The model suggests that a mixed offer. i.e., the mixture of cash payment and bidder firm's company shares, in some cases, requires the lowest threshold. It contributes to the literature that provides the theoretical justification for a mixed offer. The mix of the takeover offer that requires the lowest threshold is also derived and provides useful insights for real world situations.

### 3.1 Introduction

The most important decisions to maximize shareholder value from mergers and acquisitions, apart from strategic aspects and the integration of the companies, are the timing of the transaction in order to obtain a good price and the choice of the medium of exchange, i. e. cash, shares or a combination of both. In this chapter I will develop a real options model which addresses both of these crucial issues, allowing us to derive conditions under which a takeover deal should be observed and the structure of a takeover bid.

Recent years have seen a number of real options models of mergers and acquisitions. The vast majority of these models either assume asymmetric information about the valuation of the companies involved or the existence of synergy as a

motivation for takeovers. The focus of the following model will be on the existence of synergy from takeovers, which is not assumed to come from a particular source, although they might include economies of scale or increased operational efficiency. The size of the synergy will determine whether a takeover offer is made by the bidder and whether the target accepts this offer.

The second aspect of the offer, apart from the price which is driven by the synergy effect, is the medium of exchange. Payment to the target shareholders can be in cash, in shares of the bidder firm or a combination of them. The literature on the medium of exchange in general assumes that shares are mispriced or bidders and targets have asymmetric information about their values. These differences in valuations then determine the choice between cash and share offers, where overvalued shares are usually offered to target shareholders. For a more detailed literature review please refer to the former chapter. None of these models, however, is addressing the question of the medium of exchange when the only motivation for a takeover is the exploitation of synergy and the existing models usually do not allow for offers to consist of a mixture of cash and shares.

In this chapter these two aspects will be brought into a single real options framework which analyzes the synergy required to justify a takeover as well as the influence the payment form has on this requirement.

## 3.2 Why payment form matters

Before the model goes further into the mathematic detail, I feel it necessary to lay out a few important assumptions and illustrate the rationale of the payment form choices. At first, it is widely discussed in the literature that in a perfect market with symmetric information, no taxes and no transaction costs, the choice of the medium of the payment is irrelevant (i.e., the Modigliani-Miller theorem). Therefore, the studies in takeover payment form investigate the situations where market is realistically not perfect. For example, with asymmetric information between the takeover participants, a share offer might be a signal that bidder firm's shares are overvalued and bidder firm is attempting to take advantage of this overvaluation.

The model also considers an imperfect market where payment form choice is relevant. It is worth mentioning two important assumptions that justify the relevance of the payment form:

- It is assumed that cash and shares are not freely convertible.
- The bidder firm is assumed to have all the bargaining power so that it can

make a take-it-or-leave-it offer to the target firm.

The first statement essentially assumes a certain degree of market friction. It can be the case that the transaction cost which includes market impact is high so the convertibility between cash and share is constrained. If the target firm receives the payment all in shares and attempts to convert them to cash immediately, the market impact will be simply too high to make the transaction feasible. The first assumption makes the analysis also suitable for deals between unlisted firms. Under this assumption, for example, target firm cannot simply cash out its received bidder firm's shares or use the cash payment paid by the bidder to acquire shares of the bidding firm. Put differently, takeover participants are "stuck" with the payment they receive and any takeover decisions should be based on the payment form they receive.

It is also assumed that both bidder and target have complete information about each firm's stochastic property and the potential synergy effect that can be produced by the deal. However, the bidder firm is assumed to have all the bargaining power and therefore is able to make it take-it-or-leave-it offer. It is hence rational for the target to make its takeover decision purely basing on the real options evaluation, regardless of the existence of the synergy effect.

One of the most important implications of the real options analysis is that uncertainty affects decisions. When one is offered cash against cash, as there is no uncertainty involved (one pound of cash will always have the same value as another pound), the traditional NPV analysis can provide a simple rule to the decision: as long as what one receives is not less than what one gives up, a deal can go through.

The situation of a takeover target that sells its own business is more complicated than that. The value of the business of the target firm is assumed to follow a random process, i.e, its value is not constant over time. This random process presents a certain degree of uncertainty for the target firm when it is offered some cash to buy its business. Real options analysis suggests that in this situation, target firm will need to require more than what only justifies a non-negative NPV. The uncertainty of the target firm's value represents a possibility that target firm will obtain more profit in the future by deferring the deal. This option-like feature must be compensated for an extra premium. For example, for one pound of currently valued asset, target might require one and half pound cash to justify giving up its chance to sell the firm at better terms in the future.

What even complicates the analysis is that the takeover payment form can be made not just by cash. A form of shares or a mixture of cash and shares is not uncommon in the practice. What drives the decisions of both bidder and target

is then the uncertainty arising from the exchange of two parts: the target firm business and what bidder firm pays. Interestingly, this deal uncertainty is not simply the combination of the uncertainty of two firms' business. It is in fact the uncertainty associated with the difference of two businesses. One can look at this example. One is offered one share of a business which has a random value over time (i.e., it has uncertainty over its value) against one share of an identical business. Apparently, both sides of this deal per se have a certain level of uncertainty associated with its random value however the deal itself does not involve any change of uncertainty - the payoff from the deal will be certain at zero regardless how this particular business changes value over time. To summarize, it is the change of the uncertainty that drives the decisions of both sides and this situation can be analyzed by MARGRABE (1978)'s exchange options framework.

It can be concluded from the above analysis that payment forms with different mixture of cash and shares can lead to varying levels of change of uncertainty faced by bidder and target and in turn affect the takeover decisions of both participants. This essentially justifies why payment form matters in my theoretical framework. Then it should be further discussed why a particular payment form is used in the takeover deals.

### 3.3 The bidder's optimal payment form

It has been discussed above payment choices influence both bidder and target's takeover decisions on their thresholds. The complete information assumption essentially suggests bidder firm, with all the bargaining power, is rational to make a takeover bid with the payment form that maximizes the payoff of its own, assuming free of any agency problems.

Bidder's profit from the takeover deal is basically the value enhancement from the synergy effect minus the profit taken by the target firm. Bidder firm understands that target firm's profit is purely driven by its takeover options value. The target firm is assumed to have no bargaining power so as long as its minimum requirement - the value of option of waiting - is justified, it will accept the deal. Then it is logical for bidder to offer a payment form that minimizes the real options of target so the rest of the profit captured by bidder is maximized.

Nevertheless, the bidder's decision is still constrained by one important factor. The existing synergy effect given exogenously must be large enough to justify both bidder and target firm's real options to merge. This has been discussed in the previous chapter as a synergy effect threshold  $\alpha_{min}$ . The bidder firm's decision is in fact affected by these two values. The illustration of the idea can be

assisted by Figure (3.1). Please note that it is shown further down that payment form does affect the real options value for each participant.

The upper blue curve represents the synergy effect threshold which as discussed reflects a combined value of both participants' real options. This is the minimum requirement of the synergy effect for one to observe a deal. The curvature suggests that this requirement varies with the cash/shares mix. Please note the impact of the payment form on the synergy effect threshold can be taken in different ways as suggested by Figure (3.2) so the current setting is for illustration only. The lower dashed red curve gives the value of the target firm's takeover option in dependence of different payment form choices. There are four different scenarios worth of discussing.

- The synergy effect is at the level of Synergy A - Synergy effect A is lower than what is required to justify both bidder and target's real options for any cash/shares mix. In this case, the optimal strategy for the bidder is not to make an offer at all. The bidder needs to justify its own real option first to be able to make an offer. However it understands that the rest of the synergy after taking its own portion will not be sufficient enough for the target to accept the offer so there is no reason to make an offer.
- The synergy effect is at the level of Synergy B - Synergy effect is now large enough to make the deal go through, but only with a certain level of cash/shares mix - the point B. Certainly, it is the bidder's interests to choose a payment from that lowers target firm's options value and therefore its portion of the synergy. However, when the payment form moves right from B to D, the synergy effect threshold cannot be met any more suggesting that no deal can be observed for such circumstances. The optimal strategy for the bidder is then to offer a payment suggested by B noting that this is the only way to make a deal be accepted by both participants and a no-action will lose the opportunity to gain from the deal.
- The synergy effect is at the level of Synergy C - Bidder firm now has more choices. It understands that any payment form choices between point C1 and C2 will make the takeover happen. Then it is reasonable for it to minimize the synergy effect captured by the target. Noting that C2 is the payment mix that minimizes target firm's options value within the range of  $[C1, C2]$ , C2 is the optimal payment form that bidder should offer in the deal.
- The synergy effect is at the level of Synergy D - There is no constraint by the synergy effect threshold now. Any payment choices can be chosen to

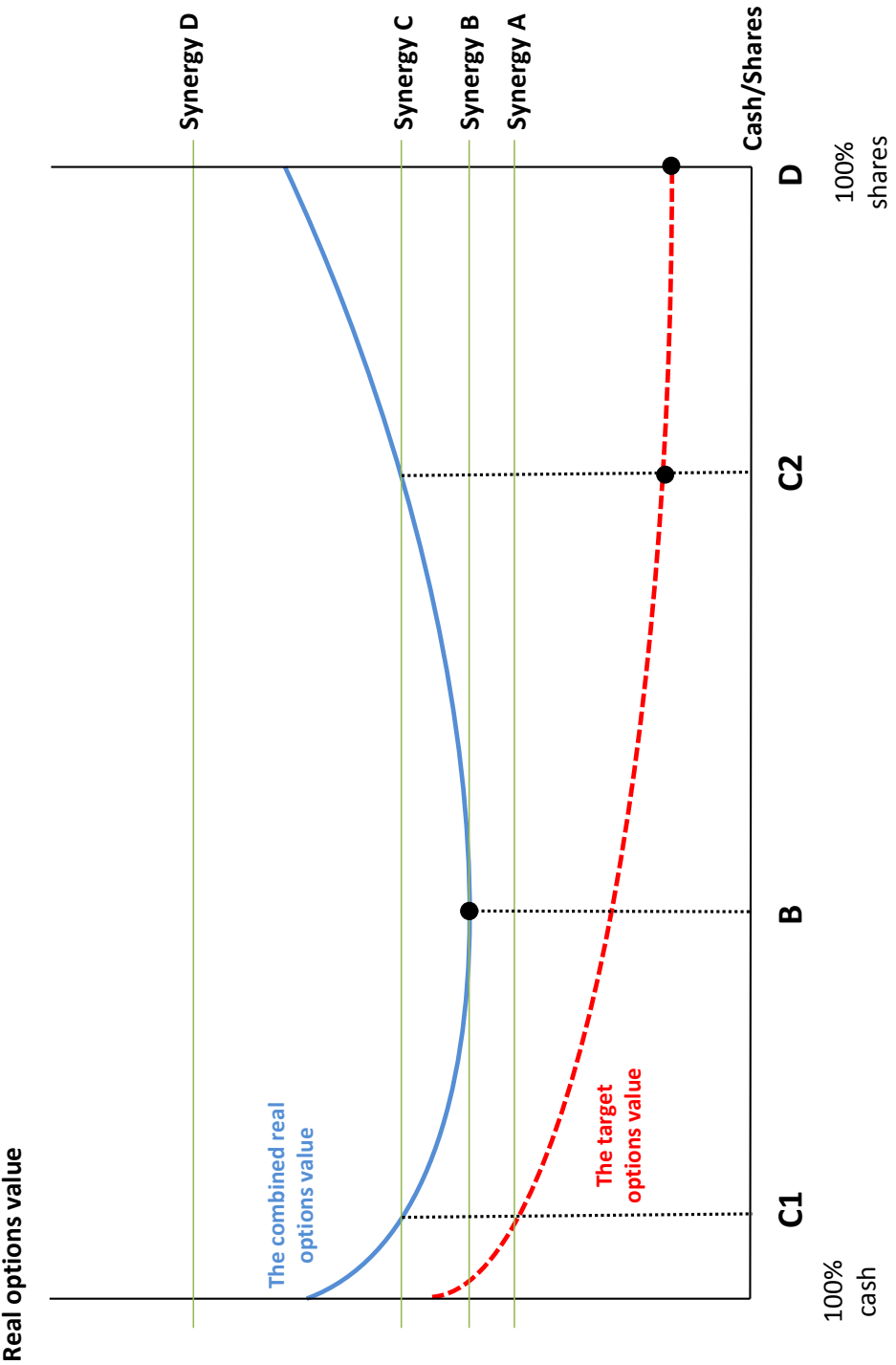


Fig. 3.1: Bidder firm's optimal strategy



make the deal go through. The decision for the bidder is fairly simple now. With all the bargaining power, it can offer 100% shares (point D) and take a significant portion of the existing synergy, leaving target firm only a small fraction.

The above analysis suggests the payment form choices selected by the bidder are affected by the synergy effect available to the deal. The following analysis will focus on the payment choice suggested by point B.

The merger of any two firms will either create or destroy value. Under perfect information, if there exist any synergy effects between any two firms that are above synergy B, two firms should have already merged. Therefore the starting point of synergy effect should be below B, for example at synergy A. Synergy effect is assumed to change value when economy grows and it is assumed that this change is continuous over time, i.e., it cannot jump from A to C or D. It is further assumed that bidder has the incentive to make an offer as soon as the deal becomes feasible, i.e., when the synergy effect between two firms reaches the threshold suggested by synergy B, one can observe a merger. According to the analysis laid out above, the payment form choice for any deal should be the one that makes the deal feasible at the first place and this payment form is defined and discussed throughout the thesis as the payment form threshold because it indicates the payment form choice when the synergy effect threshold is met at the first time.

### 3.4 Model assumptions

The previous chapter has laid out the ground for some assumptions that will be used throughout all the models in the thesis. The following model will continue to use these assumptions unless they are stated otherwise.

Consider a bidder firm ( $B$ ) which is facing an investment opportunity to take over a target firm ( $T$ ). Both firms are assumed to have total fundamental values  $S_B$  and  $S_T$  respectively, which can be viewed as the present value of their respective future uncertain cash flows. To be in line with standard literature, those fundamental values are assumed to follow geometric Brownian motions:

$$(3.1) \quad dS_i = \mu_{S_i} S_i dt + \sigma_{S_i} S_i dz_i, \quad i = B, T.$$

The correlation coefficient between the two sources of uncertainty  $S_B$  and  $S_T$  is defined to be constant, equal to  $\rho$ :

$$(3.2) \quad \varepsilon[dz_B dz_T] = \rho dt.$$

It is assumed that both equations (3.1) and (3.2) are open information to both bidder and target.

It is widely discussed that the synergy effect is one of the major reasons driving the takeover activity. It is further assumed that bidder believes that it is able to generate a certain amount of synergy through the deal, denoted by  $\alpha \cdot S_T$ .

Let  $X$  denote the value of takeover offer. The takeover transaction is now driven by two simple investment decisions: from the prospective of bidder shareholders, their potential takeover surplus will be  $(1 + \alpha)S_T - X$ ; from the prospective of target shareholders, their potential takeover surplus will be  $X - S_T$ . As discussed in the previous chapter, although target firm has complete information regarding the potential synergy effect, the bidder firm is assumed to have all the bargaining power and therefore is able to make it take-it-or-leave-it offer. Under these assumptions, it is hence rational for the target to make the takeover decision purely based on the real options optimization, regardless of the existence of the synergy effect.

As discussed in the previous chapter, a takeover threshold, denoted by  $\alpha_{min}$  can be obtained through analyzing the takeover options value of both participants.

$$(3.3) \quad \alpha \geq \alpha_{min} = \frac{F^B + F^T}{S_T} > 0.$$

It is well known that the value of real options is correlated with the level of uncertainty. The same decision maker would make different investment decisions facing different levels of uncertainty. An obvious impact from different payment methods is that each payment form results in different levels of uncertainty. For instance, in a pure cash offer, since cash has constant value, the only source of uncertainty is the fundamental value of the target firm. When a takeover offer consists of a certain amount of company shares<sup>1</sup>, there are two sources of uncertainty: bidder firm's shares and target firm's shares. As a result, the values of  $F^B$  and  $F^T$  vary with different payment methods and synergy effect threshold may have a different value in each case, too.

The competitive bid is not considered in the model. It is also necessary to distinguish the model from stochastic bargaining power model, like CRIPPS (1998), which considers that in order to reach an agreement, both buyer and seller of the goods give up their respective option value to wait. It is assumed that both bidder and target determine their respective takeover strategy in a relative isolated circumstance. For instance, a bidder firm submits its takeover bid whenever

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<sup>1</sup> It can be a mixed offer or a pure share offer.

it thinks the market condition is favorable enough, i.e., its immediate takeover payoff exceeds the option value to wait. On the other hand, target makes its takeover decision purely depending on a standard real options optimization. If the instantaneous takeover surplus exceeds the option value to wait for a better offer, target will accept the offer and one then can observe a takeover. Otherwise, target will choose to reject the offer and keep the waiting option alive.

Another important assumption is made as previously discussed that cash and shares of both firms are not freely convertible. For example, target firm cannot simply convert its received bidder's shares to cash immediately after the deal. Under this assumption, both firms need to take into account the uncertainty introduced by different payment forms and make their takeover decisions accordingly.

### 3.5 Model development

The takeover bid consists of two parts: some amount of bidder firm's company shares  $xS_B$  and some cash payment  $C$ . It is obvious that if  $x = 0$  the takeover bid is equivalent to a pure cash payment. On the other hand, if  $C = 0$ , the takeover bid is equivalent to a pure shares offer.

The analysis starts from examining an optimization strategy for both firms. The bidder is trying to maximize the value of its takeover option, which depends on the net payoff from exchanging the value of takeover bid  $X = xS_B + C$  for the target firm  $(1 + \alpha)S_T$ , which includes the synergy effects. Correspondingly the target gives up its own firm  $S_T$  for the value of takeover offer bidder submits, on which its takeover option value depends.

The model will fall into a standard exchange option category, discussed in detail in the literature review chapter, as long as the term  $X = xS_B + C$  has an explicit form of randomness. The insights gained from the literature of the spread option<sup>2</sup> help achieve this objective. A spread option is an option written on the difference of two underlying assets, the value of which normally follows a stochastic process, such as a geometric Brownian Motion. In order to obtain the future uncertain difference of two underlying assets, the owner of the option needs to pay a pre-specified price, also known as the strike or exercise price. An analogy can be made between a financial spread option and the investment opportunity of a takeover participant.

The takeover surplus the bidder is expecting at the time of exercise can be shown

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<sup>2</sup> CARMONA AND DURRLEMAN (2003) provide a good review on spread option.

as  $(1 + \alpha)S_T - xS_B - C$  ( It can be proved that  $(1 + \alpha)S_T$  follows the exactly same random process as  $S_T$ <sup>3</sup> and  $xS_B$  as  $S_B$ . One can also find out the takeover surplus resembles a payoff from a spread option, in which the owner, i.e., the bidder, has an opportunity to pay an exercise price  $C$  for the difference of two stochastic assets  $(1 + \alpha)S_T - xS_B$ . Therefore the rationale to price a financial spread option can also be applied to solve the optimization strategy of a takeover participant here.

Most results obtained from spread option pricing problem are rather complicated and hardly analytically, due to the fact that the difference of two lognormal distribution is not lognormally distributed. The idea that KIRK (1995) put forth, however, sheds a lot of lights on my study.

KIRK (1995) points out that a spread option can be examined within an exchange option framework, where an analytical solution is provided by MARGRABE (1978). Essentially he suggests to treat the cash payment and the second stochastic asset as one “combined” random process and the drift and diffusion parameters can be determined according to the relative size of random asset and cash payment. A constant can be treated as a “stochastic variable” that has the drift and diffusion parameter that are both zero. Therefore, as an approximation, the combination of a random asset and a constant value asset should have the drift and diffusion parameters that depend on the value weight of its component.

Let  $\kappa$  denote the the fraction of payments made in shares in a mixed offer upon the time of takeover (it is therefore not a stochastic variable):

$$(3.4) \quad \kappa X^* = xS_B^*,$$

$$(3.5) \quad (1 - \kappa)X^* = C.$$

Please note that  $S_B^*$  and  $X^*$  refer to the value of  $S_B$  and  $X$  upon the time of the takeover respectively.

Obviously the value of  $\kappa$  should lie between 0 and 1<sup>4</sup>. As a result of approximation, the takeover bid  $X$  follows the geometric Brownian motion given by

$$(3.6) \quad dX = \mu_X X dt + \sigma_X X dZ_X,$$

where  $\mu_X = \kappa\mu_{S_B}$ ,  $\sigma_X = \kappa\sigma_{S_B}$  and  $Z_X = Z_B$ .

<sup>3</sup> A very simple derivation of stochastic differential equation will lead to the result. It is known that  $dS_T = \mu_{S_T} S_T dt + \sigma_{S_T} S_T dz$ . Multiplying  $(1 + \alpha)$  by both sides of the equation generates  $d(1 + \alpha)S_T = \mu_{S_T}(1 + \alpha)S_T dt + \sigma_{S_T}(1 + \alpha)S_T dz$ . Let  $\hat{S}_T$  denote  $(1 + \alpha)S_T$ , it should follow the random process indicated by  $d\hat{S}_T = \mu_{S_T}\hat{S}_T dt + \sigma_{S_T}\hat{S}_T dz$ .

<sup>4</sup> The case when  $\kappa$  equals 0 or 1 actually represents a pure cash or a pure share offer, respectively.

It should be mentioned the method above only works as an approximation and readers should therefore note the limitation of this approach. A session (A.14) in the appendix is dedicated to show numerically the level of the approximation errors, which enables me to conclude that for most cases, the approximation error is small and the impact on the results is limited.

Given that approximation provides a fair estimate for most cases, the following analysis fall into a standard exchange option analysis. It can be furthermore proved that the correlation coefficient between  $S_T$  and  $X$  remains at  $\rho$ . As  $\rho_{X,S_T} = \frac{Cov(X,S_T)}{\sigma_X \sigma_{S_T}}$  and noting  $\mu_X = \frac{1}{\kappa} \mu_{S_B}$  and  $\sigma_X = \frac{1}{\kappa} \sigma_{S_B}$ , it can be concluded that  $\rho_{X,S_T} = \frac{Cov(S_B,S_T)}{\sigma_{S_B} \sigma_{S_T}} = \rho$ .

The results are given as follows:

**Lemma 4.** *Define  $S_T^*$  as the fundamental value of target at time of takeover, the value of takeover option for bidder is given by*

$$(3.7) \quad F^B = [(1 + \alpha)S_T^* - X^*] \left( \frac{X/S_T}{R^B} \right)^{\beta_B} \quad \text{for } X/S_T > R^B,$$

$$(3.8) \quad = (1 + \alpha)S_T - X \quad \text{for } X/S_T \leq R^B.$$

The takeover threshold for bidder is given by

$$(3.9) \quad R^B = \frac{\beta_B}{\beta_B - 1} (1 + \alpha),$$

where  $\beta_B$  has the value of

$$(3.10) \quad \beta_B = \frac{1}{2} - \frac{\kappa \mu_{S_B} - \mu_{S_T}}{\kappa^2 \sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\kappa \rho \sigma_{S_B} \sigma_{S_T}} - \sqrt{\left( \frac{\kappa \mu_{S_B} - \mu_{S_T}}{\kappa^2 \sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\kappa \rho \sigma_{S_B} \sigma_{S_T}} - \frac{1}{2} \right)^2 + \frac{2(r - \mu_{S_T})}{\kappa^2 \sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\kappa \rho \sigma_{S_B} \sigma_{S_T}}} < 0$$

The takeover threshold implies that if the ratio of  $X/S_T$  is less than  $R^B$ , it is optimal for bidder to enter a takeover deal.

*Proof.* See appendix. □

**Lemma 5.** *Define  $S_T^*$  as the fundamental value of target at time of takeover, the value of takeover option for target is given by*

$$(3.11) \quad F^T = [X^* - S_T^*] \left( \frac{X/S_T}{R^T} \right)^{\beta_T} \quad \text{for } X/S_T < R^T,$$

$$(3.12) \quad = X - S_T \quad \text{for } X/S_T \geq R^T.$$

The takeover threshold for target is

$$(3.13) \quad R^T = \frac{\beta_T}{\beta_T - 1},$$

where  $\beta_T$  is has the value of

$$(3.14) \quad \beta_T = \frac{1}{2} - \frac{\kappa\mu_{S_B} - \mu_{S_T}}{\kappa^2\sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\kappa\rho\sigma_{S_B}\sigma_{S_T}} + \sqrt{\left(\frac{\kappa\mu_{S_B} - \mu_{S_T}}{\kappa^2\sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\kappa\rho\sigma_{S_B}\sigma_{S_T}} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_{S_T})}{\kappa^2\sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\kappa\rho\sigma_{S_B}\sigma_{S_T}}} > 1$$

The takeover threshold implies that if the ratio of  $X/S_T$  is greater than  $R^T$ , it is optimal for target to accept a takeover offer.

*Proof.* See appendix.  $\square$

Given the individual takeover threshold for bidder and target, the synergy effect threshold to justify a takeover deal can therefore be derived. It should fulfill the minimum requirements for both bidder and target instantaneously.

**Lemma 6.** Consider a takeover transaction where the payment is made by partial cash and partial share. The synergy effect threshold, denoted by  $\alpha_{min}^M$ , which justifies the option value to wait for both bidder and target of the deal, is given by

$$(3.15) \quad \alpha \geq \alpha_{min}^M = \frac{\beta_T}{\beta_T - 1} \frac{\beta_B - 1}{\beta_B} - 1,$$

where  $\beta_{B,T}$  are aforementioned. Noting that  $\beta_T > 1$  and  $\beta_B < 0$ ,  $\alpha_{min}^M$  must be positive.

*Proof.* In order to justify a takeover transaction, the following rationale must be fulfilled: the maximum value the bidder is willing to offer must be not less than the minimum value that is able to drive the target to accept the offer. In mathematical terms, the following two conditions must hold in the same time:  $X/S_T \leq \frac{\beta_B}{\beta_B - 1}(1 + \alpha)$  and  $X/S_T \geq \frac{\beta_T}{\beta_T - 1}$ . It suggest that  $\frac{\beta_B}{\beta_B - 1}(1 + \alpha) \geq \frac{\beta_T}{\beta_T - 1}$ . Rearranging it provides the results shown above.  $\square$

The result indicates that the synergy effect threshold depends on a variety of variables:  $\kappa$  (the fraction ratio of shares),  $\mu_{S_B}$ ,  $\mu_{S_T}$  (expected growth rates of both bidder and target's fundamental values),  $\sigma_{S_B}$ ,  $\sigma_{S_T}$  (volatilities of bidder and target firm),  $\rho$  (the correlation between two stochastic processes) and  $r$  (the discount rate).

### 3.6 Characteristics of the takeover threshold

The synergy effect threshold derived above imposes a minimum requirement on the synergy effect for a takeover deal to be observed under the current model setting. In this section, the characteristics of this threshold will be examined.

It starts with the proving the convergence of case for  $\kappa = 0, 1$ .

Intuitively, the case  $\kappa = 1$  means that the takeover bid consists of 100 % of bidder firm's company shares, i.e.,  $X$  is equivalent to  $S_B$ . The takeover decision is still subject to an exchange option analysis and therefore the results obtained converge to a pure share situation.

The case  $\kappa = 0$ , however, makes the proof a bit more difficult. In a pure cash offer, the payment  $X$  has a constant value over time and therefore equation(3.6) is no longer appropriate. The proof starts with a standard real option analysis with only one random variable involved (i.e., the model discussed in the previous chapter) and compare the result with what is derived from the model and it has been approved that they are identical. Please refer to the appendix for further detail.

To summarize with the following lemma:

**Lemma 7.** *Let  $\alpha_{min}^C$  and  $\alpha_{min}^S$  denote the synergy effect threshold in a takeover deal where a pure cash payment or a pure shares offer is provided respectively.*

$$(3.16) \quad \alpha_{min}^M = \alpha_{min}^C \quad \text{when } \kappa \equiv 0$$

$$(3.17) \quad \alpha_{min}^M = \alpha_{min}^S \quad \text{when } \kappa \equiv 1$$

*Proof.* See appendix. □

In the rest of the section, the first order derivative of the synergy effect threshold ( $\alpha_{min}^M$ ) will be derived to conduct a sensitivity analysis. Essentially the synergy effect threshold is resulted from a combination of two effects: the takeover options premiums from both bidder and target. However, it is worth mentioning that this combined effect is not simply the linear combination of two respective option premium.

Furthermore, it is clear that  $\alpha_{min}^M$  is strongly related to the takeover options premiums and therefore should share a lot of similarities with the feature of a standard real options premium.

The results are shown in the following lemmas.

**Lemma 8.**

$$(3.18) \quad \frac{\partial \alpha_{min}^M}{\partial r} < 0$$

*Proof.* See appendix.  $\square$

It is commonly discussed in the real options literature (e.g., DIXIT AND PINDYCK (1994)) that a higher discount rate induces a lower value of option to wait. A higher discount rate results in a greater discount effect of forgone cash flows from delaying the project and therefore favors the decision to take the immediate payoff. This theory applies in both call and put options values. As a result one should be expecting a negative correlation between the discount rate and the value of the synergy effect threshold.

**Lemma 9.** *Let*

$$\sigma_M \equiv \sqrt{\kappa^2 \sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\rho\kappa\sigma_{S_B}\sigma_{S_T}},$$

$$\Pi_M \equiv \left( \frac{\kappa\mu_{S_B} - \mu_{S_T}}{\kappa^2 \sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\rho\kappa\sigma_{S_B}\sigma_{S_T}} - \frac{1}{2} \right)^2 + \frac{2(r - \mu_{S_T})}{\kappa^2 \sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\rho\kappa\sigma_{S_B}\sigma_{S_T}}.$$

*Then*

$$(3.19) \quad \frac{\partial \alpha_{min}^M}{\partial \mu_{S_T}} < 0, \quad \text{If } \frac{[(1 + 2\sqrt{\Pi_M})\sigma_M^2 + 2\mu_{S_B} - 2\mu_{S_T}](\sigma_M^2 - 2\mu_{S_B} + 2\mu_{S_T})}{(-1 + 2\sqrt{\Pi_M})\sigma_M^2 - 2\mu_{S_B} + 2\mu_{S_T}} < 0,$$

$$(3.20) \quad \frac{\partial \alpha_{min}^M}{\partial \mu_{S_T}} > 0, \quad \text{If } \frac{[(1 + 2\sqrt{\Pi_M})\sigma_M^2 + 2\mu_{S_B} - 2\mu_{S_T}](\sigma_M^2 - 2\mu_{S_B} + 2\mu_{S_T})}{(-1 + 2\sqrt{\Pi_M})\sigma_M^2 - 2\mu_{S_B} + 2\mu_{S_T}} > 0.$$

*Proof.* See appendix.  $\square$

Lemma 9 investigates how the takeover threshold interacts with the growth rate of the target firm. To interpret the rationale of the fairly complicated conditions seen above, let me start from a simpler situation, where only cash is involved in a takeover deal. When a bidder is acquiring a target with cash, its call options value increases when the expected growth rate of the target firm increases. The rationale is not hard to explain. A higher growth rate for the target firm, with everything else being equal, makes the waiting more worthwhile (the value for the target firm is more likely to be higher in the future) and therefore increases the option value for the bidder firm. On the other side, the target firm sees its takeover options value decrease when the expected growth rate of its own firm is higher. The target firm essentially holds a put option on its own asset and a high growth rate reduces this put option value and the target firm therefore favors no-waiting in this scenario.

While the exchanging of two random assets make the analysis a bit more complicated, it follows the exact same rationale as discussed above. As  $\mu_T$  increases the bidder will have a relatively lower growth rate, which will result in the value



of the bidder company to increase relatively slowly. Thus expected benefits from exercising the option at a later point of time are lower and the prospects of immediate exercise to take advantage of the synergy effect are becoming more important. Thus the option value for the target reduces and thereby reducing the synergies required to make an offer. For the bidder the opposite is true, the higher growth rate of the target will provide the bidder with an incentive to postpone the acceptance of a bid, increasing the option value and increasing the synergy level at which an offer will be accepted. When looking at the synergy threshold at which an offer is accepted, its change will depend on the relative strength of these two opposing effects. Essentially (3.19) gives the conditions when the effect of a changing  $\mu_{S_T}$  on the target real options value is larger than that on the bidder real options value and hence an increasing  $\mu_{S_T}$  reduces the synergy effect threshold. (3.20) provides the conditions when the effect of a changing  $\mu_{S_T}$  on the target real options value is less than that on the bidder real options. Consequently, when the expected growth rate of the target firm increases, the synergy effect threshold becomes higher.

**Lemma 10.**

$$(3.21) \quad \frac{\partial \alpha_{min}^M}{\partial \mu_{S_B}} < 0, \quad \text{If } \frac{[(1 + 2\sqrt{\Pi_M})\sigma_M^2 - 2\kappa\mu_{S_B} + 2\mu_{S_T}](\sigma_M^2 + 2\kappa\mu_{S_B} - 2\mu_{S_T})}{(-1 + 2\sqrt{\Pi_M})\sigma_M^2 + 2\kappa\mu_{S_B} - 2\mu_{S_T}} < 0,$$

$$(3.22) \quad \frac{\partial \alpha_{min}^M}{\partial \mu_{S_B}} > 0, \quad \text{If } \frac{[(1 + 2\sqrt{\Pi_M})\sigma_M^2 - 2\kappa\mu_{S_B} + 2\mu_{S_T}](\sigma_M^2 + 2\kappa\mu_{S_B} - 2\mu_{S_T})}{(-1 + 2\sqrt{\Pi_M})\sigma_M^2 + 2\kappa\mu_{S_B} - 2\mu_{S_T}} > 0.$$

*Proof.* See appendix. □

The analysis for lemma 10 can be conducted in a similar fashion. When bidder firm's expected growth rate becomes higher, with everything else being equal, it reduces its own takeover options value but enhances the takeover options value for the target, with the same rationale explained in lemma 9. Again, the synergy effect threshold  $\alpha_{min}^M$  is a combination of two opposing effects. The combined effect therefore relies on the relative power of two effects. Conditions that the decrease in the bidder firm's takeover options value will more than offset the increase in the target firm's takeover options value by per unit of increase in the expected growth rate of the bidder is given by (3.21). As a result, a fast growing bidding firm will in fact reduce the synergy effect threshold. The opposite scenario is specified by condition (3.22).

**Lemma 11.**

$$(3.23) \quad \frac{\partial \alpha_{min}^M}{\partial \sigma_{S_T}} < 0, \quad \text{If } \sigma_{S_T} < \rho \kappa \sigma_{S_B},$$

$$(3.24) \quad \frac{\partial \alpha_{min}^M}{\partial \sigma_{S_T}} > 0, \quad \text{If } \sigma_{S_T} > \rho \kappa \sigma_{S_B}.$$

*Proof.* See appendix.  $\square$

**Lemma 12.**

$$(3.25) \quad \frac{\partial \alpha_{min}^M}{\partial \sigma_{S_B}} < 0, \quad \text{If } \kappa \sigma_{S_B} < \rho \sigma_{S_T},$$

$$(3.26) \quad \frac{\partial \alpha_{min}^M}{\partial \sigma_{S_B}} > 0, \quad \text{If } \kappa \sigma_{S_B} > \rho \sigma_{S_T}.$$

*Proof.* See appendix.  $\square$

It is widely discussed in real options literature (e.g., DIXIT AND PINDYCK (1994)) that volatility/uncertainty delays investment decision. While shown in a different form, the same rationale applies in the analysis for this lemma. Essentially the takeover deal involves the exchange of two random assets: payment  $X$  with the volatility of  $\kappa \sigma_{S_B}$  and target firm's business with the volatility of  $\sigma_{S_B}$ . The volatility of the takeover deal in essence has a form given by  $\sigma_M$ , which has the value of  $\sigma_M = \sqrt{\kappa^2 \sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\rho \kappa \sigma_{S_B} \sigma_{S_T}}$ . What simplifies the analysis is both the bidder and target faces the same exchange volatility, i.e., when  $\sigma_M$  increases, the value of both bidder and target firm's options value will increase and therefore the synergy effect threshold will increase. Having said that, the analysis now boils down to investigating how the change of  $\sigma_{S_B}$  and  $\sigma_{S_T}$  affects the value of  $\sigma_M$ . With the above analysis in mind, the results in the Lemma 11 and 12 are not difficult to understand. Under the condition  $\sigma_{S_T} < \rho \kappa \sigma_{S_B}$ , an increase in the target firm's volatility will in fact reduces the total volatility in the deal and therefore reduces the synergy effect threshold and when  $\sigma_{S_T} > \rho \kappa \sigma_{S_B}$ , an increase in the target firm's volatility will increase the uncertainty/volatility of the deal and therefore increase the threshold. Lemma 12 can be analyzed in the similar fashion.

Apart from  $\kappa^5$ , there is only one parameter yet to discuss, which is the correlation coefficient  $\rho$ . A simple argument tells that a higher correlation coefficient means that both sides of exchanging assets have more similarity in terms of randomness, and therefore the total volatility of the exchanging deal should be reduced. The

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<sup>5</sup> Analyzing  $\kappa$  generates the most important implication of the model, which will be conducted in an individual section.

insinuation is simply to put down. A higher correlation between two exchange assets means the lower uncertainty involved with the exchange deal and therefore drive down the takeover threshold of both side and therefore the total takeover threshold.

**Lemma 13.**

$$(3.27) \quad \frac{\partial \alpha_{min}^M}{\partial \rho} < 0$$

*Proof.* See appendix. □

As DIXIT AND PINDYCK (1994) explain, holding the variances fixed, a greater correlation between the changes in two exchange assets implies less uncertainty over their ratio, and hence a reduced incentive to wait.

### 3.7 The payment form threshold

The parameter  $\kappa$  plays a crucial role in the analysis. As shown in lemma 7, the model results apply to a pure cash offer deal when  $\kappa = 0$  and a pure share offer deal when  $\kappa = 1$ .

As discussed above, the synergy effect threshold is the minimum requirement for a takeover to be observed. The lower the threshold, the higher the chance that a takeover will emerge for each of the takeover participant to realize the synergy effect. In this section, the model explores the payment form threshold that requires the lowest value of the synergy effect threshold. This payment form threshold will in turn maximize the probability of realizing value creation through corporate takeover deals.

Noting that  $\kappa$  is within the interval  $[0, 1]$ , it can be concluded that pure cash offer results in the lowest takeover threshold, i.e., the lowest  $\alpha_{min}^M$ , if one can prove that  $\alpha_{min}^M$  is a monotonic increasing function of  $\kappa \in [0, 1]$ . On the other hand, if  $\alpha_{min}^M$  is a monotonic decreasing function of  $\kappa \in [0, 1]$ , pure shares, i.e.,  $\kappa = 1$ , should be the payment form threshold. Clearly, mixed offer will be observed if a certain value of  $\kappa \in (0, 1)$  minimizes the  $\alpha_{min}^M$ .

This section conducts an analysis on the synergy effect threshold  $\alpha_{min}^M$  in dependence of  $\kappa$ . The results suggest that it is possible to require less synergy effect than the other two payment forms for a mixed offer. The results are summarized in the following lemma:

**Lemma 14.** *Define the value of  $\kappa_1$  and  $\kappa_2$  as follows:*

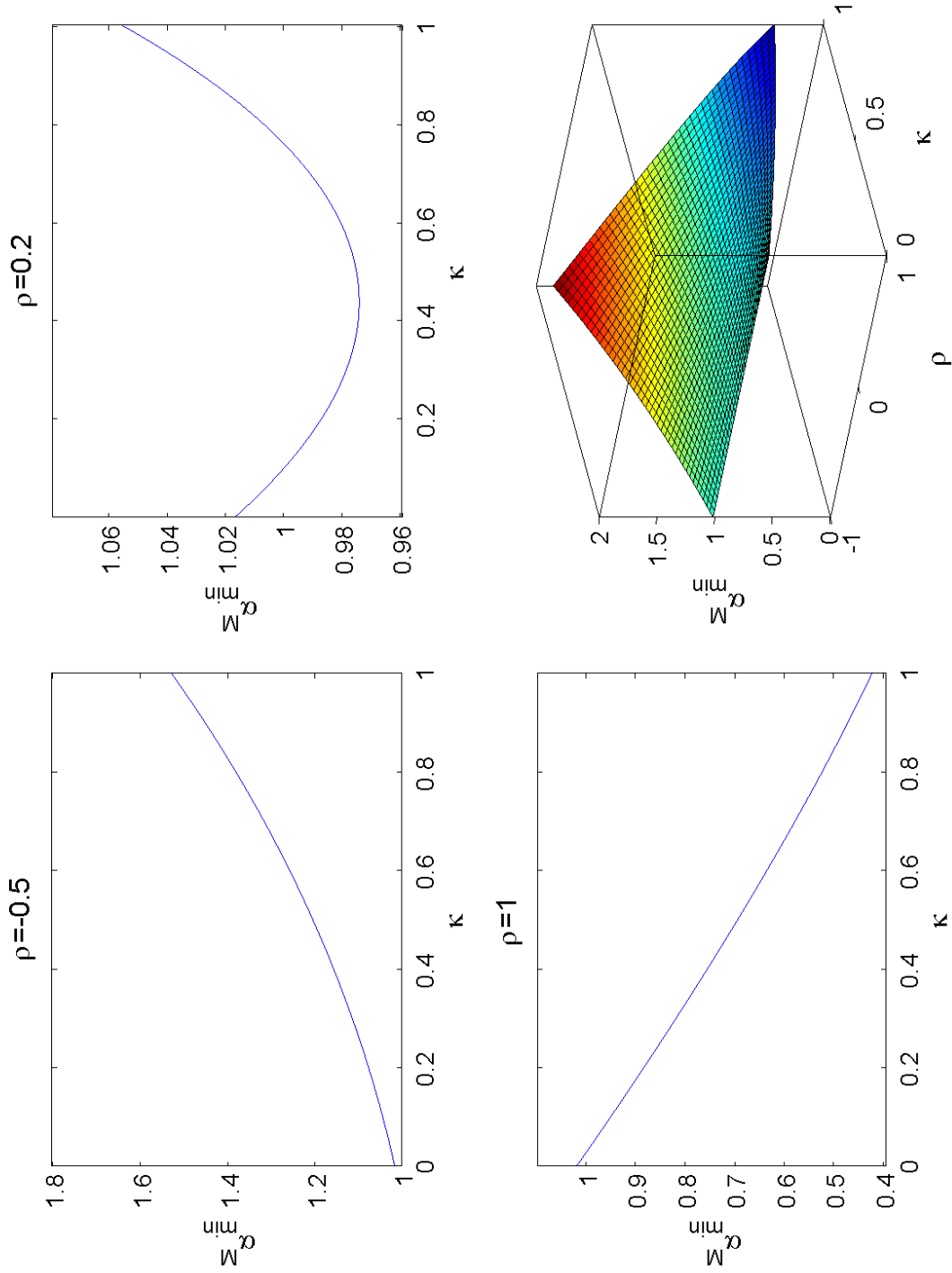
$$(3.28) \quad \kappa_{1,2} \equiv \frac{\mu_{S_B}^2 + 2r\sigma_{S_B}^2 + \rho\mu_{S_B}\sigma_{S_B}\sigma_{S_T}}{3\mu_{S_B}\sigma_{S_B}^2} \pm \frac{\sqrt{(\mu_{S_B}^2 + 2r\sigma_{S_B}^2 + \rho\mu_{S_B}\sigma_{S_B}\sigma_{S_T})^2 + 3\mu_{S_B}\sigma_{S_B}^2[\mu_{S_B}(\sigma_{S_T}^2 - 2\mu_{S_T}) - 4r\rho\sigma_{S_B}\sigma_{S_T}]}}{3\mu_{S_B}\sigma_{S_B}^2}.$$

*The rule to determine the payment form threshold should be given by the following statements.*

1. *If  $\kappa_1 > 1 > 0 > \kappa_2$ , then  $\alpha_{min}^M$  is a monotonic increasing function of  $\kappa$  during the interval  $[0, 1]$  which indicates that pure cash offer is the payment form that requires the lowest synergy effect threshold.*
2. *If  $\kappa_1 > \kappa_2 > 1$  then  $\alpha_{min}^M$  is a monotonic decreasing function of  $\kappa$  during the interval  $[0, 1]$  which indicates that pure share offer is the payment form that requires the lowest synergy effect threshold.*
3. *If  $\kappa_1 > 1 > \kappa_2 > 0$ , then  $\alpha_{min}^M$  is a decreasing function of  $\kappa$  during the interval  $[0, \kappa_2]$  and an increasing function of  $\kappa$  during the interval  $[\kappa_2, 1]$  which indicates that partial cash and partial shares is the payment form that requires the lowest synergy effect threshold. The value of threshold  $\kappa$  is given by  $\kappa_2$ .*

*Proof.* See appendix. □

An illustrative numerical example should help understand the above proposition. Consider a base with the parameter value given by  $\mu_{S_B} = \mu_{S_T} = 0.01$ ,  $\sigma_{S_B} = 0.05$ ,  $\sigma_{S_T} = 0.1$ ,  $r = 0.05$ . In figure 3.2, it is shown numerically the synergy effect threshold in dependence of different correlation coefficients. When  $\rho = -0.5$ , the value of  $\kappa_1 = 9.2336$  and  $\kappa_2 = -0.5770$  i.e.  $\kappa_2 < 0$ . According to the first criteria of the lemma,  $\alpha_{min}^M$  is increasing when  $\kappa$  increases, the pure cash offer has the lowest required synergy. When  $\rho = 1$ ,  $\kappa_1 = 9.0452$  and  $\kappa_2 = 1.6215$ . Since  $\kappa_2 > 1$ , the lemma suggests a negative relationship between  $\alpha_{min}^M$  and  $\kappa$  which is consistent with what numerical example represents. Under this market condition, pure shares offer requires the least synergy effect threshold. When  $\rho$  has a value 0.2, it is easy to obtain that  $\kappa_1 = 9.1635$  and  $\kappa_2 = 0.4352$ . As a result,  $\alpha_{min}^M$  hits its minimum value within the range  $\kappa \in [0, 1]$  at the point  $\kappa = \kappa_2 = 0.4352$ . The illustrative example demonstrates numerically what the proposition puts forward, as shown by the top right sub-figure in figure 3.2. In this case, the payment form threshold should consist of 43.52% of company shares

Fig. 3.2:  $\alpha_{\min}^M$  in dependence of  $\kappa$

of the bidder firm and 56.46% of the cash payment because this offer requires the lowest possible synergy effect to justify a takeover transaction.

It can be observed further from the above lemma that the characteristics of each firm's randomness ( $\mu_{S_B}$ ,  $\mu_{S_T}$ ,  $\sigma_{S_B}$ ,  $\sigma_{S_T}$ ) have an impact on the payment form threshold. Besides, the discount factor ( $r$ ) and the correlation coefficient between the two randomness ( $\rho$ ) also affect it. The non-linear feature of equation (3.28) makes it very difficult, if not impossible, to conduct an analytical sensitivity analysis on the results. Therefore a numerical analysis is shown in Figures 3.3 and 3.4.

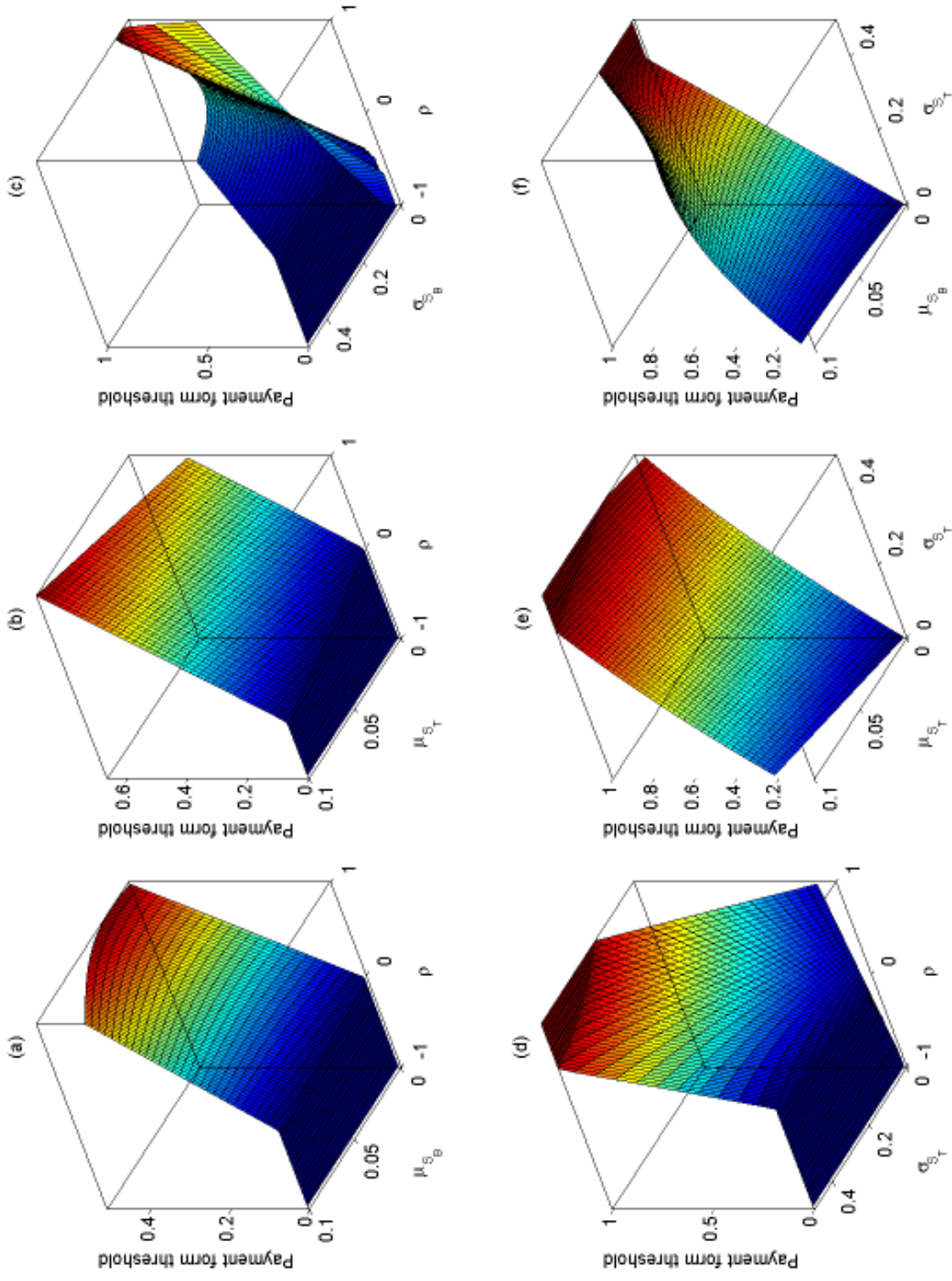
**Lemma 15.** *The partial derivatives of  $\kappa_{\min}$ , which is defined as the payment form threshold that leads to the lowest required takeover threshold, are given by*

- $\frac{\partial \kappa_{\min}}{\partial \rho} > 0$
- $\frac{\partial \kappa_{\min}}{\partial \mu_{S_T}} > 0$
- $\frac{\partial \kappa_{\min}}{\partial \sigma_{S_T}} > 0$
- $\frac{\partial \kappa_{\min}}{\partial \mu_{S_B}} < 0$  except for the conditions that  $\sigma_{S_T}$  has a relatively low value (the interval close to 0) or  $\mu_{S_T}$  has a relatively high value (the interval close to 0.1)
- $\frac{\partial \kappa_{\min}}{\partial \sigma_{S_B}} > 0$  except for the conditions that  $\sigma_{S_T}$  has a relatively high value (the interval close to 0.5) or  $\mu_{S_T}$  has a relatively low value (the interval close to 0)

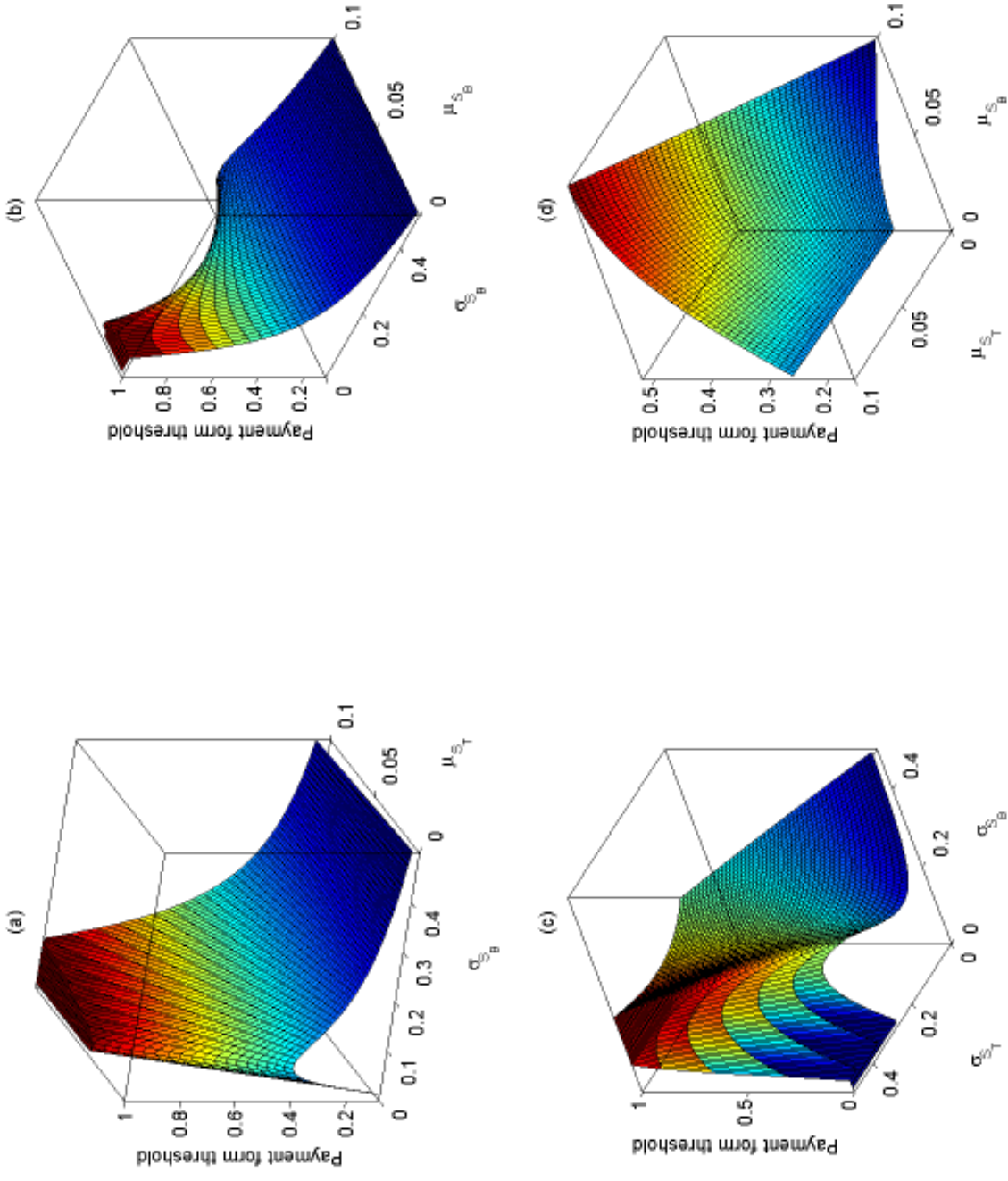
*The above properties represent those parameter constellations which can be deemed to be most relevant in applications.*

It is clear that the higher the correlation coefficient between the two firms, the more the similarity they share in terms of their stochastic property. As discussed above, a higher correlation between two firms reduce the uncertainty of the deal, which in turn drives down the options value of both participants. As a result, when the correlation coefficient is high, a higher proportion of the shares in the mixed offer is more likely to require a lower value of synergy effect threshold and therefore fulfils the requirement of the payment form threshold.

The lemma also shows that a fast growing target firm or a volatile target firm tends to result in a higher proportion of shares used in the takeover offer to reduce the required synergy effect. To be able to understand the rationale behind these results, one needs to explore more how these two parameters interact with the takeover threshold in a pure cash offer. In a pure cash offer, as bidder firm's shares



**Fig. 3.3:** The payment form threshold in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_{S_T} = 0.02$ ,  $\mu_{S_B} = 0.04$ ,  $\sigma_{S_T} = 0.1$ ,  $\sigma_{S_B} = 0.2$ ,  $\rho = 0.5$ ,  $r = 0.25$ .



**Fig. 3.4:** The payment form threshold in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_{S_T} = 0.02$ ,  $\mu_{S_B} = 0.04$ ,  $\sigma_{S_T} = 0.1$ ,  $\sigma_{S_B} = 0.2$ ,  $\rho = 0.5$ ,  $r = 0.25$ .



is not involved in the deal, the only source of the uncertainty is from the target firm. The volatility of the deal essentially is in proportion with the volatility of the target firm. To summarize, one should always observe a higher takeover threshold with a higher volatility of the target firm. Target firm's expected growth rate, as discussed in the previous lemma, however tends to have opposing impacts on target and bidder firm's takeover options value. Fortunately, for a pure cash offer, it can be approved that the impact on the bidder firm's options value will always more than offset the impact on the target firm's options value. Said differently, an increasing  $\mu_{S_T}$  will always increase the takeover threshold in a pure cash situation (Mathematical proof simply requires differentiating  $\alpha_{min}^M(\kappa = 0)$  with respect to  $\mu_{S_T}$ . The result will be always positive). However, as shown in the previous lemmas,  $\mu_{S_T}$  and  $\sigma_{S_T}$  can in many cases effectively reduce the takeover threshold when bidder's business is part of the deal. In order to reduce the synergy effect threshold, it is then not difficult to understand that a higher expected growth rate or a higher volatility of the target firm increases the attractiveness of the shares in a mixed offer.

The impact of bidder firm's characteristics ( $\mu_{S_B}$  and  $\sigma_{S_B}$ ) on the payment choice are, however, more ambiguous. As one can observe, an increasing proportion of shares is resulted by either a higher value of  $\mu_{S_B}$  when  $\mu_{S_T}$  is relatively high or a lower value of  $\mu_{S_B}$  when  $\mu_{S_T}$  is relatively low. The impact of  $\sigma_{S_B}$  on the payment form choice is also unclear. In some cases when  $\sigma_{S_T}$  or  $\mu_{S_T}$  is small, a higher volatility of bidding firm seems to favor the choice of cash in the mixed offer. In other situations, it has an opposite impact. Due to the ambiguous results, the further empirical analysis will only focus on the results the provide clear implications.

This lemma can now be used to test the results of the model empirically by comparing the medium of exchange for various takeovers. To summarize, the takeover offer has an increasing fraction of share payment if volatility of the target increases, target performance (expected growth rate) increases, or correlation of the two companies increases.

As most of the existing literature investigating the medium of exchange has focused on the announcement returns and post-takeover performance, the coming section will also investigate these aspects in my model.

### 3.8 Division of tender offer benefits

Apart from analyzing the payment form threshold, it is also interesting to analyze the size and division of benefits between the bidder and target. The expected

announcement returns of the bidder and target should be affected by the synergy effect as well as the the medium of exchange being employed. The returns to the bidder and target,  $Re^B$  and  $Re^T$ , are given as follows:

$$(3.29) \quad Re^B = \frac{(1 + \alpha_{min}^M)S_T^* - X^*}{S_B^*} = \frac{S_T^*}{S_B^*} \left( 1 + \alpha_{min}^M - \frac{\beta_T}{\beta_T - 1} \right),$$

$$(3.30) \quad Re^T = \frac{X^* - S_T^*}{S_T^*} = \frac{1}{\beta_T - 1}.$$

Furthermore, given the total takeover benefits being  $\alpha_{min}^M S_T$ , the fraction of surplus accrued to the target, denoted by  $f_T$ , should have the value of

$$(3.31) \quad f_T = \frac{X^* - S_T^*}{\alpha S_T^*} = \frac{Re^T}{\alpha_{min}^M}.$$

Figures 3.5 -3.9 numerically demonstrate the return to bidder and target and division of surplus between the two participants in dependence of some relative parameters.

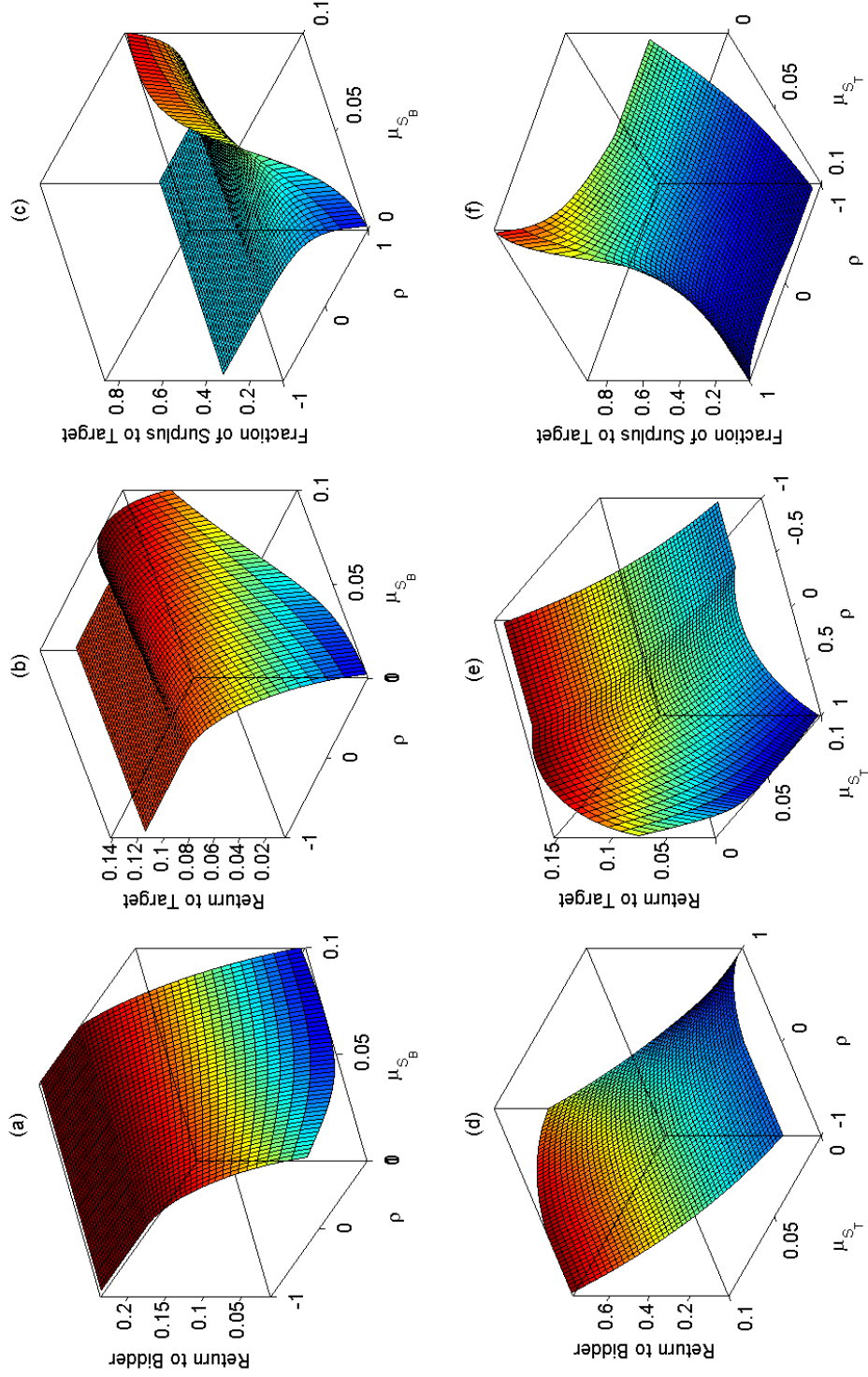
One might ask since the takeover deal results from the exercise of the real options, does the return change in the same pattern of the value of option? The answer is that it is only partially correct. As an illustration, one can take a look at the correlation coefficient effect on the return.

It is noted that a higher correlation will result in a stronger diversification effect and therefore reduce the total risk involved in an exchange option. As a result, a decreasing return with respect to correlation should be expected. However, as one can observe from figure 3.5 - (b), this is not the case. There is a non-linear relationship between return to target and correlation especially when  $\mu_{S_B}$  tends to be high. The explanation to this inconsistency with standard real options analysis can be laid out as follows. The change of correlation coefficient, apart from affecting the diversification effect, has an impact on the payment form threshold, which in turn, via  $\kappa$ , affects the real options values. The combination of these two effects affects the impact from a correlation coefficient. As a result it is not always true that a small correlation leads to a higher return.

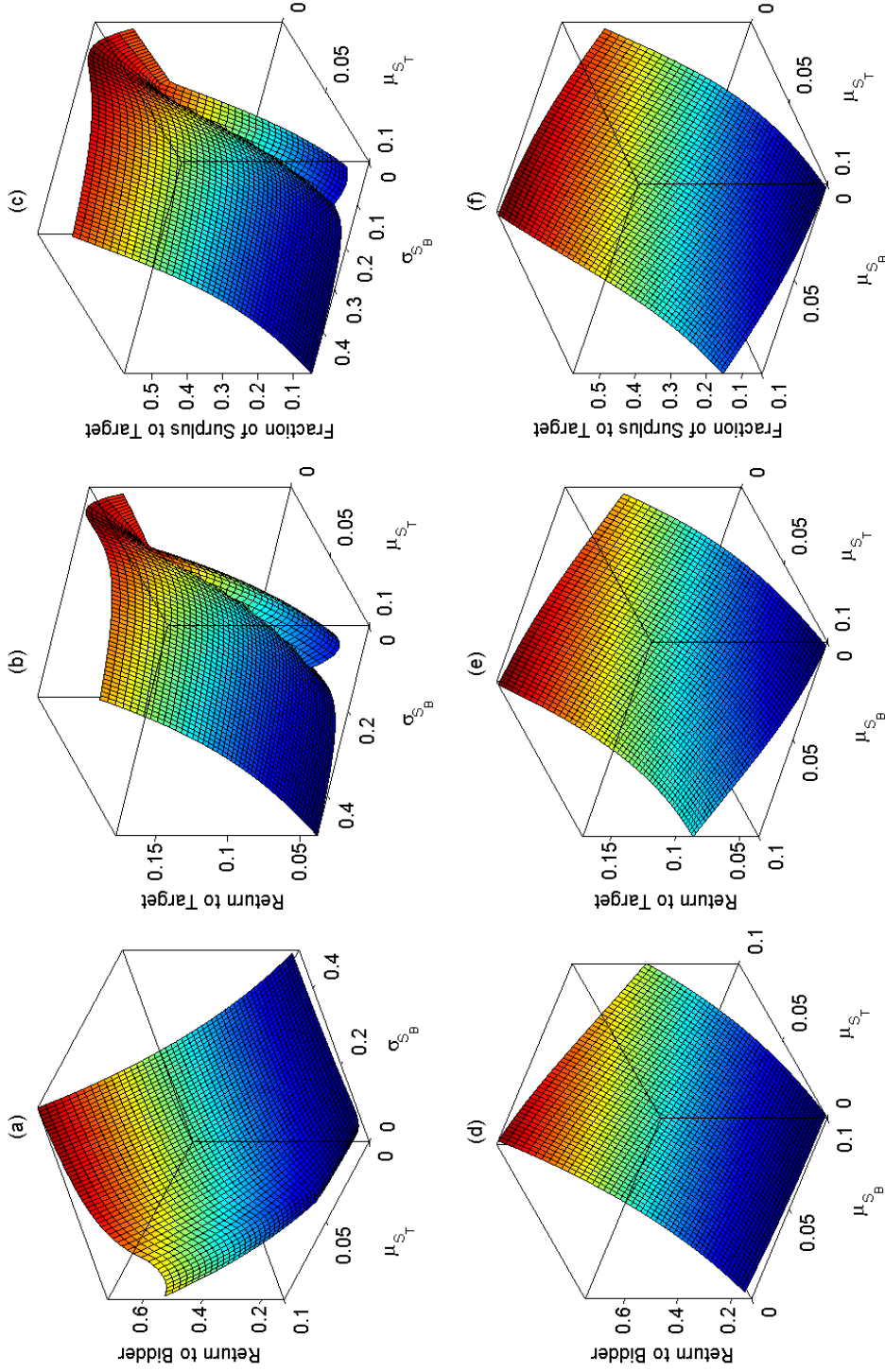
I can now summarize the main results obtained from the numerical analysis. Since the trends for some parameters are too unclear to provide any useful implications, I only lay out the ones that provide a relatively clear cut.

**Lemma 16.** *The partial derivatives of the bidder returns are*

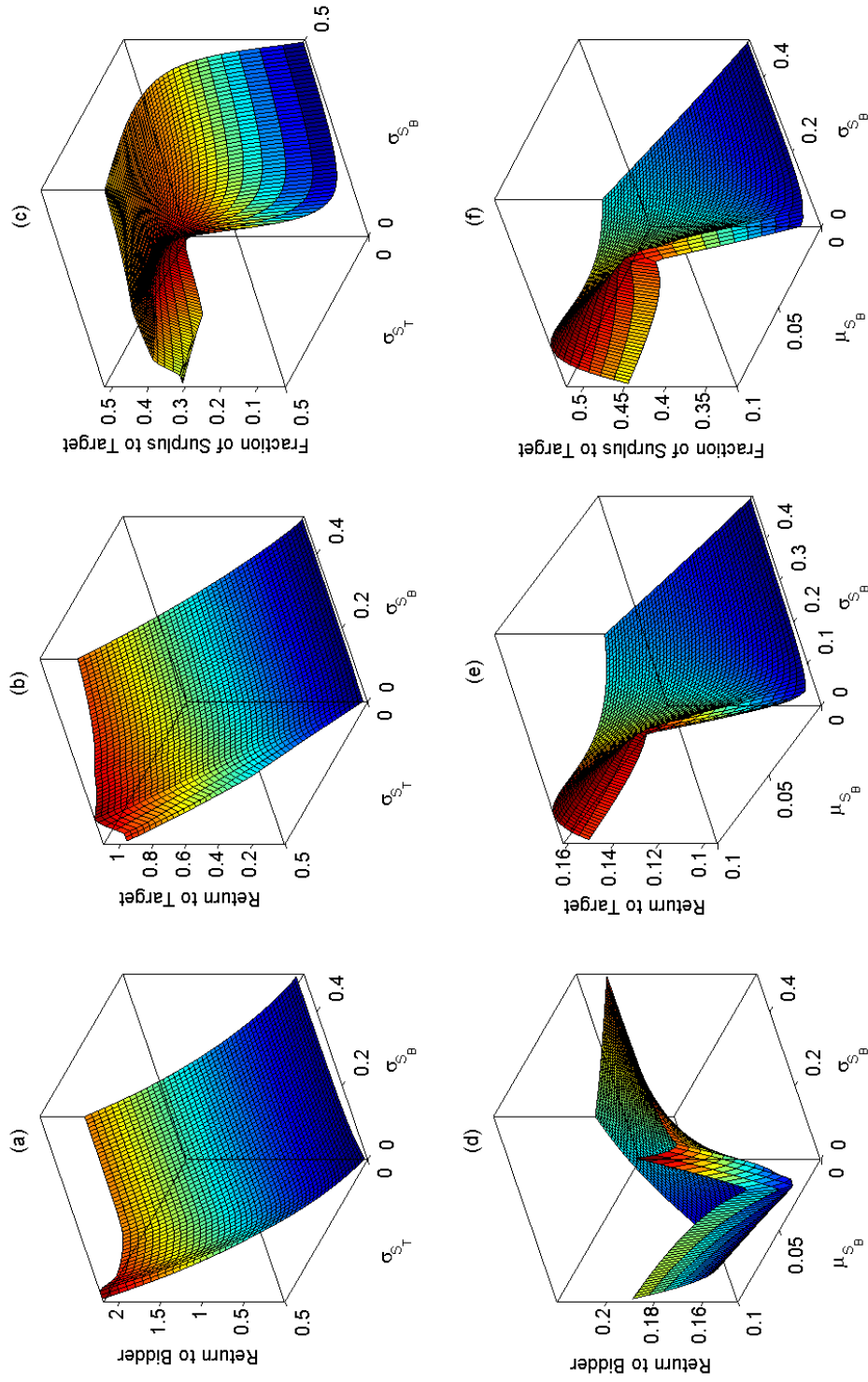
- $\frac{\partial Re^B}{\partial \mu_{S_T}} > 0,$



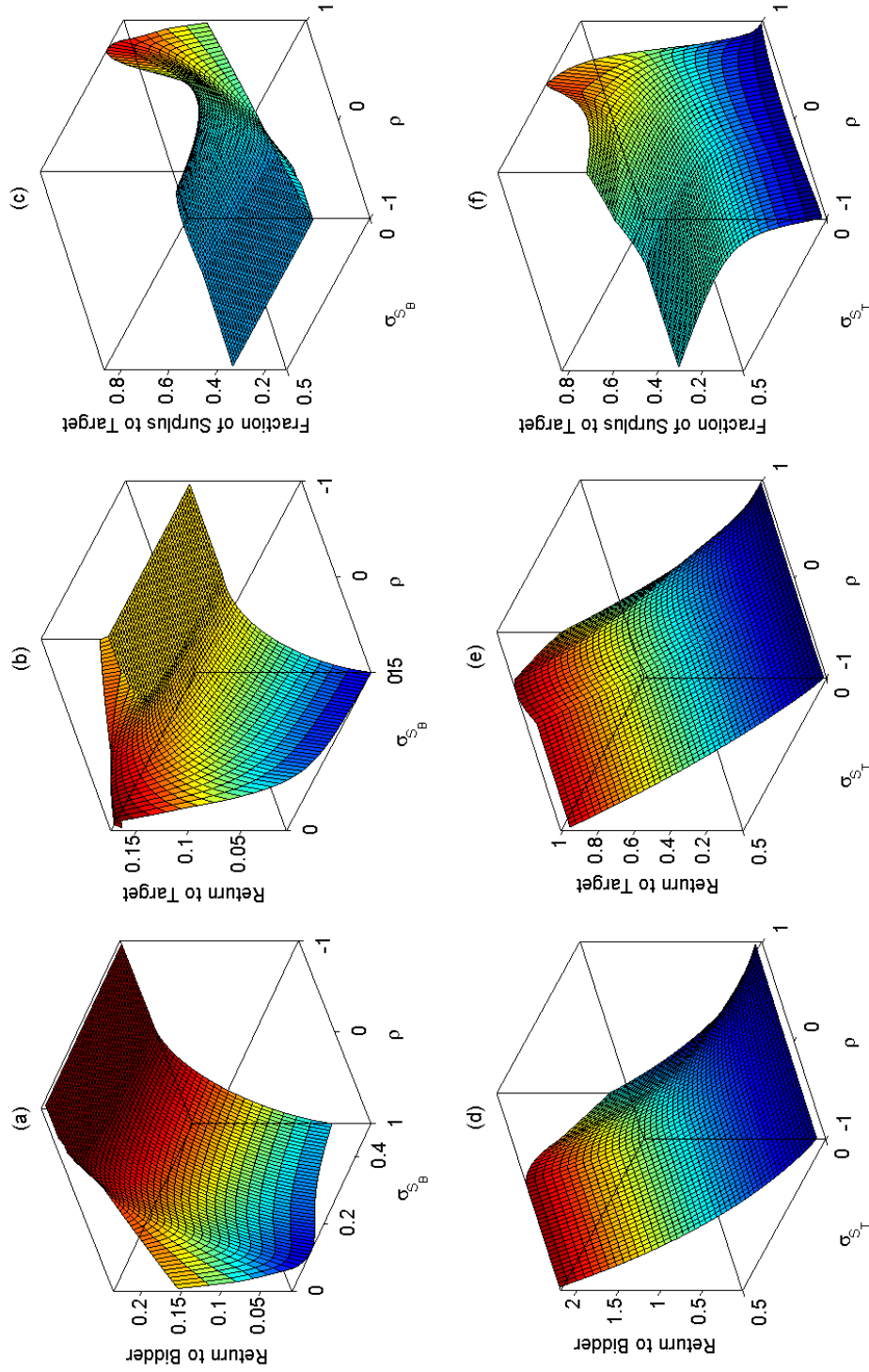
**Fig. 3.5:** The return to bidder and target as well as the fraction of surplus to target in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_{S_T} = 0.02$ ,  $\mu_{S_B} = 0.04$ ,  $\sigma_{S_T} = 0.1$ ,  $\sigma_{S_B} = 0.2$ ,  $\rho = 0.5$ ,  $r = 0.25$ .



**Fig. 3.6:** The return to bidder and target as well as the fraction of surplus to target in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_{S_T} = 0.02$ ,  $\mu_{S_B} = 0.04$ ,  $\sigma_{S_T} = 0.1$ ,  $\sigma_{S_B} = 0.2$ ,  $\rho = 0.5$ ,  $r = 0.25$ .

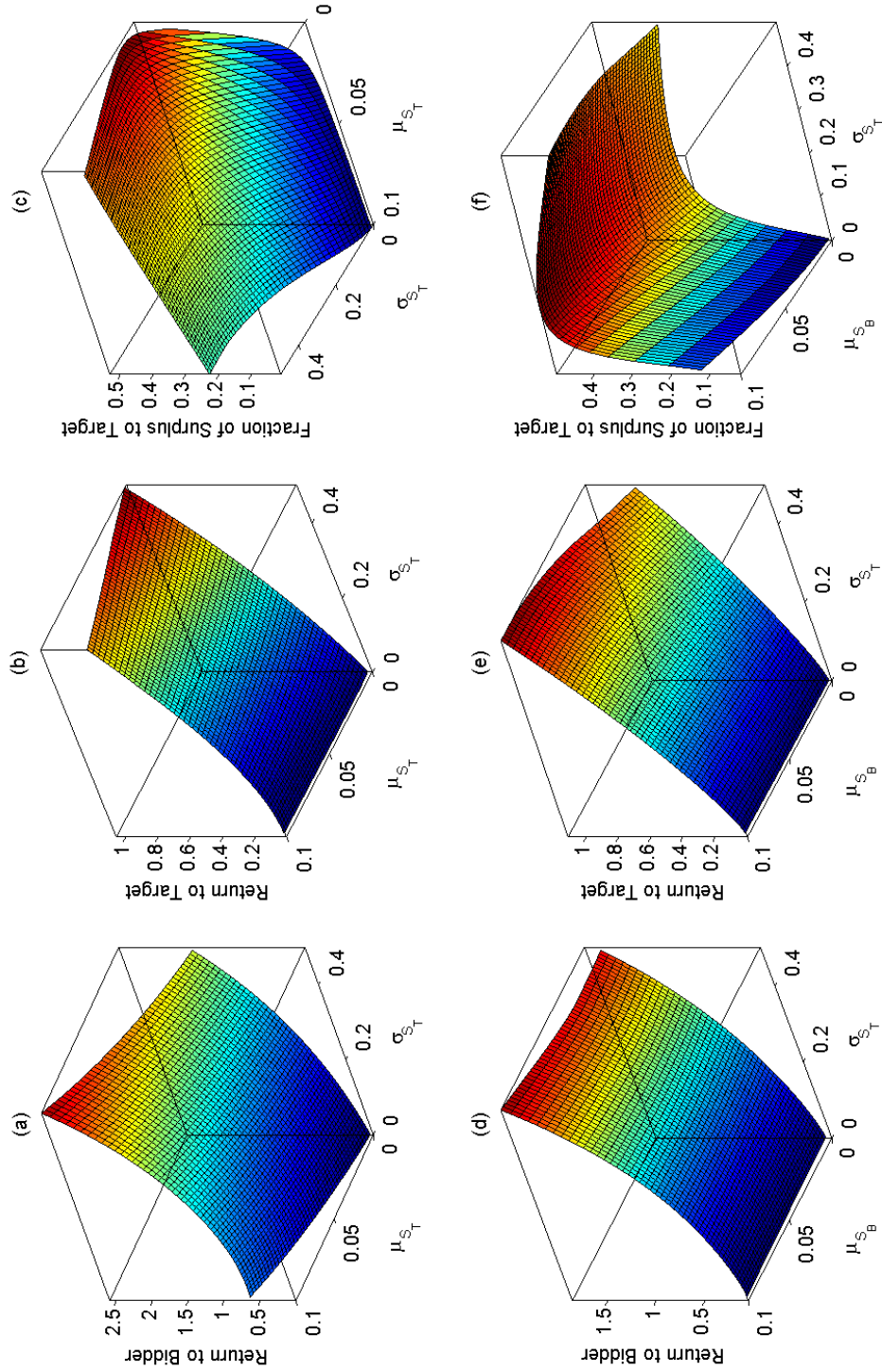


**Fig. 3.7:** The return to bidder and target as well as the fraction of surplus to target in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_{S_T} = 0.02$ ,  $\mu_{S_B} = 0.04$ ,  $\sigma_{S_T} = 0.1$ ,  $\sigma_{S_B} = 0.2$ ,  $\rho = 0.5$ ,  $r = 0.25$ .



**Fig. 3.8:** The return to bidder and target as well as the fraction of surplus to target in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_{S_T} = 0.02$ ,  $\mu_{S_B} = 0.04$ ,  $\sigma_{S_T} = 0.1$ ,  $\sigma_{S_B} = 0.2$ ,  $\rho = 0.5$ ,  $r = 0.25$ .





**Fig. 3.9:** The return to bidder and target as well as the fraction of surplus to target in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_{S_T} = 0.02$ ,  $\mu_{S_B} = 0.04$ ,  $\sigma_{S_T} = 0.1$ ,  $\sigma_{S_B} = 0.2$ ,  $\rho = 0.5$ ,  $r = 0.25$ .

- $\frac{\partial Re^B}{\partial \sigma_{S_T}} > 0$ ,
- $\frac{\partial Re^B}{\partial \mu_{S_B}} < 0$ , except for the conditions that  $\sigma_{S_B}$  has a relative low value (the interval close to 0) or  $\sigma_{S_B}$  has a relatively high value (the interval close to 0.5)
- $\frac{\partial Re^B}{\partial \rho} < 0$ .

*The results on the volatility of the bidder,  $\sigma_{S_B}$ , are affected by the restriction that  $\kappa \in [0; 1]$ , such that relationships are sometimes not monotonic. The above properties represent those parameter constellations which can be deemed to be the most relevant in applications.*

A higher expected growth rate of the target firm's fundamental value, a more volatile target firm or a lower correlation coefficient between the two firms result in a higher takeover return to the bidder. Apart from the conditions specified in the lemma, a lower growth term for bidder firm will have the same effect. While it has been discussed that the takeover returns are not identical to takeover options premium, it is undeniable that they should share a lot of similarities. These similarities make the interpretation of the results much easier. Essentially, the bidder exchanges its own business with those of target firm. As aforementioned, a higher expected growth rate for the target firm, a lower expected growth rate of its own firm or a lower correlation coefficient will increase the benefit of waiting therefore increasing the options premium. The announcement returns to the bidder will increase as a result. These observations are consistent with the arguments in the previous lemmas. Interestingly, it is shown that a more volatile target firm will in effect increase the options premium for the bidder firm and subsequently its takeover return. This observation provides the support that for the parameters considered in the numerical demonstration above, an increasing target firm volatility will increase the uncertainty of deal facing bidder firm, therefore requiring more returns to justify the exercise of its takeover option.

**Lemma 17.** *The partial derivatives of the target returns are*

- $\frac{\partial Re^T}{\partial \mu_{S_T}} < 0$ ,
- $\frac{\partial Re^T}{\partial \sigma_{S_T}} > 0$ ,
- $\frac{\partial Re^T}{\partial \mu_{S_B}} > 0$ ,
- $\frac{\partial Re^T}{\partial \rho} < 0$  except for the conditions that  $\sigma_{S_B}$  has a relatively low value (the interval close to 0) or  $\mu_{S_B}$  has a relative high value (the interval close to 0.1).



*The results on the volatility of the bidder,  $\sigma_{S_B}$ , are affected by the restriction that  $\kappa \in [0; 1]$ , such that relationships are sometimes not monotonic. The above properties represent those parameter constellations which can be deemed to be the most relevant in applications.*

The rationale for analyzing the target firm's return is not different. The return to target firm is negatively correlated with the expected growth rate of its own firm. A higher  $\mu_{S_T}$  accelerates the takeover decision for target when it gives up  $S_T$ , because it is unlikely that waiting will result in better deals. Target is more likely to wait and therefore the takeover option value increases when it is expecting to receive a high growth asset. A higher  $\sigma_{S_T}$  results in a higher total volatility involved in a takeover deal and therefore increases target firm's options premium. A higher correlation, however, affects total uncertainty in an opposite way and therefore reduces target returns.

The analysis then move to the distribution of takeover surplus between bidder and target, measured by  $f_T$ .

**Lemma 18.** *The partial derivatives of the fraction of takeover benefits gained by targets are*

- $\frac{\partial f^T}{\partial \mu_{S_T}} < 0$ ,
- $\frac{\partial f^T}{\partial \mu_{S_B}} > 0$ .

*The results on  $\sigma_{S_B}$ ,  $\sigma_{S_T}$ ,  $\rho$ , are affected by the restriction that  $\kappa \in [0; 1]$ , such that relationships are sometimes not monotonic. The above properties represent those parameter constellations which can be deemed to be the most relevant in applications.*

There are only two parameters that have a clear impact on the fraction of takeover surplus allocated to target. Note that  $f^T$  essentially reflects how the takeover surplus is allocated between two participants or the relative size of bidder and target firms' options premium. A higher growth term for target results in both a higher return for bidder and a lower return for target. It therefore should lead to a lower fraction of takeover benefits captured by the target. On the other hand, a higher growth rate of bidder firm brings the bidder firm a lower return and target firm a higher return. It consequently should enable the target to take an increasing fraction of the synergy effect. For other parameters, the returns are ambiguous and cannot provide a clear implication.

Furthermore, it is found that in most cases bidder firm obtains a larger return than the target, which is obviously against the widely accepted empirical findings that the vast majority of takeover benefits go to the target. This can be

explained by the model's lack of consideration of bidding competition, which normally drives asymmetric distribution of the takeover surplus. Besides, ignoring any negotiation process over the surplus, the model purely looks at the size of the real options to wait for better conditions at a later point in time. These real options, under some certain circumstances, can be higher for the bidder than the target.

### 3.9 Empirical implications

The results presented above constitute the empirical hypotheses that require further research on. I therefore provide a summary of the empirical implications generated from the above analysis.

The tender offer has an increasing fraction of share payments if

- volatility of the target increases,
- target performance (expected growth rate) increases,
- correlation of the two companies increases.

The bidder obtains an increasing return in a tender offer transaction if

- volatility of the target increases,
- target performance (expected growth rate) increases,
- correlation of the two companies decreases.

The target obtains an increasing return in a tender offer transaction if

- volatility of the target increases,
- target performance (expected growth rate) decreases,
- bidder performance (expected growth rate) increases.

The target receives a larger fraction of benefits in a tender offer transaction if

- bidder performance (expected growth rate) increases,
- target performance (expected growth rate) decreases.

### 3.10 Conclusion

In this chapter, a real options model of takeovers motivated by synergies is developed. The results show how the synergy effect required to conduct a takeover depend not only on the parameters of both firms but also on the medium of exchange. Different payment forms affect the takeover decisions by introducing different opportunity costs for both bidder and target to enter the deal. Analyzing the conditions which require the smallest synergy effect, it is obtained that in many realistic cases an offer consisting of both cash and shares is required. The criteria of determining the method of payment - the payment form threshold - is also provided. The model further discusses the characteristics of the payment form threshold and the takeover returns to both bidders and targets when that payment form is used. The model suggests that high growth target firms should result in more returns captured by bidder firms and fast growing bidders produce more takeover returns to target firms.

## Chapter 4

# The payment form threshold in mergers

### 4.1 Introduction

It is widely accepted that corporate takeovers are normally justified by positive cash flow synergy. The operational synergy is the major motive for takeover transactions and is examined by parameter  $\alpha$  in the previous chapter. In this chapter, it is my aim to consider a new structure how the takeover deal is conducted, therefore checking the consistency of the results to further test the robustness of the model.

In the previous model, a tender offer deal where the exchange of two firms is viewed on a stand-alone base. For example, when the target makes its decision, it considers 1) the stochastic process of its own firm, and 2) the stochastic process of takeover payment, which in turn depends in part on the stochastic process of the bidder firm. In this chapter, it is assumed that the stochastic property of the shares component does not only depend on the shares of the bidder firm, it also depends on the target firm. Put differently, the shares component of the payment form reflects the integration of two businesses. Therefore the target firm effectively exchanges 1) its own firm with 2) a fraction of newly created firm. As LELAND AND SKARABOT (2003) suggest, the expected growth and volatility parameters of the newly created firm depend on the growth and volatility parameters of each firm, the correlation coefficient of the two firms and the relative size of the two firms at the time of the deal.

The second new feature arises from the fact that cash payment of a takeover deal can be paid out from the newly created firm. The cash payment, if sufficiently large, can also influence the stochastic process of the merged firm. As a result, the takeover strategy should change accordingly.

Thirdly, the synergy effect is defined as a value enhancement over the newly joint firm, instead of over the target firm only. It essentially suggests that the operating synergy is from the integration of two businesses, where not only post-deal target

firm value increases as a result of the deal, the bidder firm also benefits from the transaction and hence sees its value increase as well after the deal.

However, it should be noted that these new assumptions do not suggest the invalidity of previous chapter's assumptions. The previous chapter is appropriate for deals where the size of the target is relatively small so that the impact of the merger on the bidder firm's fundamentals can be ignored. In these deals, the newly acquired target remains as a separately run business and the synergy effect is achieved through for example a better management of the new business instead of the business integration of the two firms.

Apparently, the new assumptions make the model suit better for a more typical merger deal where the size of the target firm is relative large enough to be able to have an impact on the fundamental feature of the newly created firm. However, the additional complexity of the model makes the analytical solutions very difficult, if not impossible, to obtain and therefore the analysis in the chapter is mainly numerical based.

## 4.2 Model development

The model assumptions are consistent with what is stated in the previous chapters unless otherwise discussed. Consider two companies who contemplate a takeover deal, where one acts as the potential bidder, denoted  $B$ , and the other as the target, denoted  $T$ .  $\Delta$  is defined as  $\Delta = \frac{S_T^*}{S_T^* + S_B^*}$ , which reflects the relative size of these two companies upon the time of the deal.  $S_i$ ,  $i = B, T$ , denotes the fundamental values of the bidder and the target. For consistency with the notion of target and bidder I assume without loss of generality that  $\Delta \leq \frac{1}{2}$ , i.e., the size of the target should not be larger than the bidder at the time of the deal. The two companies follow a standard geometric Brownian motion:

$$(4.1) \quad dS_i = \mu_i S_i dt + \sigma_i dz_i, \quad i = B, T,$$

where  $dz_i$  is a standard Wiener process and  $E[dz_B dz_T] = \rho dt$ .

If the bidder approaches the target it has the choice of offering either a cash payment  $X \geq 0$  or shares giving the target a fraction  $\delta \in [0, 1]$  of the newly formed company, or a combination of these two payment forms. For simplicity it is assumed that cash is available in any required quantity at no additional costs, i.e., the bidder is cash rich and therefore there are no financing activities involved. The value of the newly formed company, denoted by  $N$ , will consist of the values of the target and the bidder,  $S_B + S_T$ , reduced by the amount of cash paid out to target shareholders,  $X$ , as well as a synergy effect which is a fraction  $\alpha$  of the

joint company after the takeover:

$$(4.2) \quad S^N = (1 + \alpha)(S_B + S_T - X).$$

With this assumption it can also take into account the fact that paying out cash to the shareholders of the target company reduces the assets and thus the value of the joint company. Furthermore, it is proposed that cash payment can in fact come from the cash holding of the bidder or the target, thus allowing the cash holding of the target to be used to finance the takeover.

Further the model explicitly takes into account the reduction in the risk of the merged company due to the diversification of assets from the initially separate companies, an aspect usually not considered in the literature. It is done by interpreting the merged company as a portfolio consisting of the two original companies, in the way that LELAND AND SKARABOT (2003) suggest. In technique terms, the newly created firm has a stochastic process satisfying

$$(4.3) \quad dS^N = \mu^N S^N dt + \sigma^N S^N dz,$$

where the values of  $\mu^N$  and  $\sigma^N$  are given by the following analysis.

According to LELAND AND SKARABOT (2003), the expected growth and variance of the combination of the bidder and the target,  $S_B + S_T$  can be approximately given by

$$(4.4) \quad \mu' = \Delta\mu_T + (1 - \Delta)\mu_B,$$

$$(4.5) \quad \sigma' = [(1 - \Delta)^2\sigma_B^2 + \Delta^2\sigma_T^2 + 2\Delta(1 - \Delta)\sigma_B\sigma_T\rho]^{\frac{1}{2}},$$

where  $\Delta$  is as aforementioned.

$\kappa \equiv \frac{\delta S^{N*}}{S_T^{N*}}$  is defined as the fraction of the payment the target receives in shares upon the time of the deal (therefore  $\kappa$  is a constant). Note that  $S_T^{N*}$  represents what the target receives after the deal and is a combination of a fraction of the newly created firm and some cash payment. It therefore has a value of  $S_T^{N*} = \delta S^{N*} + X$ . Given that  $S^{N*} = (1 + \alpha)(S_B^* + S_T^* - X)$ , I can solve after inserting for the expression of the cash payment  $X$  to be

$$(4.6) \quad X = \frac{(1 - \kappa)(1 + \alpha)\delta(S_B^* + S_T^*)}{\kappa + \delta(\kappa - 1)(1 + \alpha)}.$$

Inserting this relationship it can be obtained

$$(4.7) \quad \zeta \equiv \frac{S_B^* + S_T^*}{S_B^* + S_T^* - X} = 1 + \frac{1 - \kappa}{\kappa}(1 + \alpha)\delta,$$

where  $\zeta$  specifies the effect of cash paid out to shareholders on the merged firm's stochastic property and can be viewed as a leverage effect. Again, please note

that all variables in the equation (4.6) and (4.7) refer to their values upon the time of takeover and therefore  $\zeta$  has a constant value.

Following a similar approximation method discussed in the previous chapter, the results obtained above can be used to determine the expected growth and volatility of the new company:

$$(4.8) \quad \mu^N = \zeta \mu',$$

$$(4.9) \quad \sigma^N = \zeta \sigma'.$$

In a general case where the medium of exchange consists of a combination of cash and shares the target and the bidder will after the deal receive total payments of

$$(4.10) \quad S_T^N = \delta S_N + X,$$

$$(4.11) \quad S_B^N = (1 - \delta) S_N,$$

respectively. Again, using a similar approximation method, the expected growth rates and standard deviations bidder and target shareholders receive will thus be approximately given by

$$(4.12) \quad \mu_B^N = \mu^N,$$

$$(4.13) \quad \sigma_B^N = \sigma^N,$$

$$(4.14) \quad \mu_T^N = \kappa \mu^N,$$

$$(4.15) \quad \sigma_T^N = \kappa \sigma^N.$$

What bidder firm receive is essentially a fraction of the firm  $N$ . It therefore has the same stochastic property with  $S^N$ . On the other side, target firm receives a certain amount of cash in addition to the rest of the firm. Its growth and volatility term is therefore “diluted” by the cash payment as suggested in the approximation method and its effect is captured by  $\kappa$ .

In the appendix, there is an independent session (A.15) demonstrating the magnitude of the approximation error and readers should refer to it for the limitations of this particular approximation method. However, in most cases it has been shown that approximation error is modest.

Further denoting the covariances between the returns on the initial holding of the shareholders of the bidder (target),  $S_B$  ( $S_T$ ), and its holding after the deal,  $S_B^N$  ( $S_T^N$ ), by  $\sigma_{BB}$  ( $\sigma_{TT}$ ), one can get

$$(4.16) \quad \sigma_{BB} = \left(1 + \frac{1 - \kappa}{\kappa}(1 + \alpha)\delta\right) ((1 - \Delta)\sigma_B^2 + \Delta\sigma_T\sigma_B\rho),$$

$$(4.17) \quad \sigma_{TT} = \kappa \left(1 + \frac{1 - \kappa}{\kappa}(1 + \alpha)\delta\right) (\Delta\sigma_T^2 + (1 - \Delta)\sigma_T\sigma_B\rho).$$

I can now interpret the decision of the target and the bidder to conduct a takeover as a real option on the exchange of their initial holdings of  $S_T$  and  $S_B$  for  $S_T^N$  and  $S_B^N$ , respectively. The analysis of these real options for the bidder and the target follows standard exchange real options procedures, which are similar to the ones conducted in the previous chapter. In short, bidder firm exchanges  $S_B$  for  $S_B^N$  and target firm exchanges  $S_T$  for  $S_T^N$ . The stochastic properties of all variables have been discussed above (equations 4.12 to 4.15).

**Lemma 19.** *The condition under which the target and the bidder would engage into a takeover is given as follows: A takeover can be observed iff  $\alpha \geq \alpha_{\min}$  with  $\alpha_{\min}$  implicitly defined as the solution to the equations*

$$(4.18) \quad \frac{S_B^N}{S_B} = \frac{\beta_B}{\beta_B - 1},$$

$$(4.19) \quad \frac{S_T^N}{S_T} = \frac{\beta_T}{\beta_T - 1}$$

where  $\beta_B$  and  $\beta_T$  depend on the parameters  $\mu_B$ ,  $\mu_T$ ,  $\sigma_B$ ,  $\sigma_T$ ,  $\rho$ ,  $\alpha$ ,  $\Delta$  as well as the decision variables  $\delta$  and  $X$ , and the risk-free rate  $r$ :

$$(4.20) \quad \beta_B = \frac{1}{2} - \frac{\mu_B^N - \mu_B}{\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB}} + \sqrt{\left(\frac{\mu_B^N - \mu_B}{\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB}} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_B)}{\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB}}}$$

$$(4.21) \quad \beta_T = \frac{1}{2} - \frac{\mu_T^N - \mu_T}{\sigma_T^{N^2} + \sigma_T^2 - 2\sigma_{TT}} + \sqrt{\left(\frac{\mu_T^N - \mu_T}{\sigma_T^{N^2} + \sigma_T^2 - 2\sigma_{TT}} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_T)}{\sigma_T^{N^2} + \sigma_T^2 - 2\sigma_{TT}}}$$

*Proof.* See appendix. □

Rearranging  $\Delta \equiv \frac{S_T^*}{S_B^* + S_T^*}$  gives

$$(4.22) \quad \frac{S_B^*}{S_T^*} = \frac{1 - \Delta}{\Delta}$$

$$(4.23) \quad \frac{S_T^*}{S_B^*} = \frac{\Delta}{1 - \Delta}$$

Noting  $X = \frac{(\kappa-1)(1+\alpha)\delta(S_B^*+S_T^*)}{\delta(\kappa-1)(1+\alpha)-\kappa}$  the above equations become

$$(4.24) \quad \frac{\kappa(1+\alpha)(1-\delta)}{[(\kappa-1)(1+\alpha)\delta-\kappa](\Delta-1)} = \frac{\beta_B}{\beta_B-1}$$



$$(4.25) \quad \frac{-(1+\alpha)\delta}{[(\kappa-1)(1+\alpha)\delta-\kappa]\Delta} = \frac{\beta_T}{\beta_T-1}$$

The above equations have two unknown variables - the synergy effect threshold  $\alpha$  (discussed by  $\alpha_{min}$  in the following analysis) and the allocation rule  $\delta$ . Noting that  $\beta_T$  and  $\beta_B$  are also dependent on  $\alpha$  and  $\delta$  itself is a function of  $\alpha$  it can only be solved numerically. The properties of the synergy effect threshold are analyzed in the coming section.

It should be noted that the analysis above does not apply to the case  $\kappa = 0$  as  $\kappa$  appears in the denominator for some expressions. In other words, the above analysis does not apply to a pure cash offer. Therefore further analysis is conducted below.

When a cash offer is used, the analysis for the bidder's decision remains the same as the exchange still involves two stochastic variables - its own firm and the newly created entity. However, from a target point of view, there is only one stochastic variable involved, i.e., the takeover transaction is simply exchanging a constant value cash payment and a stochastic value, the target firm's business. As a result, the exchange options analysis still applies to bidder's takeover decision while a standard real options analysis will now apply to target.

The analysis start from the target. In a pure cash offer, the target exchanges the value of its firm  $S_T$  for a cash payment  $X$  and the bidder exchanges  $S_B$  for  $S_B^N = S^N = (1+\alpha)(S_B + S_T - X)$ . The target holds a standard real put option to sell its shares at a price of  $X$  which has been discussed in Lemma 2 in Chapter 2. Applying its result one can get

$$(4.26) \quad X = \frac{\theta-1}{\theta} S_T^*,$$

$$(4.27) \quad \theta = \frac{1}{2} - \frac{\mu_T}{\sigma_T^2} - \left[ \left( \frac{1}{2} - \frac{\mu_T}{\sigma_T^2} \right)^2 + \frac{2r}{\sigma_T^2} \right]^{\frac{1}{2}}.$$

The bidder's situation can be analyzed in a similar way as a mixed offer. What bidder firm pays is the its own firm ( $S_B$ ). What bidder firm receives is the newly created firm  $S^N$  as target has no ownership in the new firm. Then it boils down to an exchange option discussed before and the analysis from equation (4.3) to (4.25) applies.

Plugging the result that  $X = \frac{\theta-1}{\theta} S_T^*$  and  $\Delta = \frac{S_T^*}{S_T^* + S_B^*}$  into equation (4.7) yields

that  $\zeta = \frac{\theta}{\theta + \Delta(1-\theta)}$ , which in turns gives

$$(4.28) \quad \mu_B^N = \frac{\theta}{\theta + \Delta(1-\theta)} \mu',$$

$$(4.29) \quad \sigma_B^N = \frac{\theta}{\theta + \Delta(1-\theta)} \sigma',$$

$$(4.30) \quad \sigma_{BB} = \frac{\theta}{\theta + \Delta(1-\theta)} [(1-\Delta)\sigma_B^2 + \Delta\sigma_B\sigma_T\rho].$$

Using these parameters one can solve for the exchange option of the bidder in the same way as in the case of  $\kappa > 0$ . Consequently, equation (4.24) becomes

$$(4.31) \quad (1 + \alpha) \left( 1 + \frac{\Delta}{\theta(1-\Delta)} \right) = \frac{\beta_B}{\beta_B - 1},$$

where  $\beta_B$  has the value given by equation (4.20).

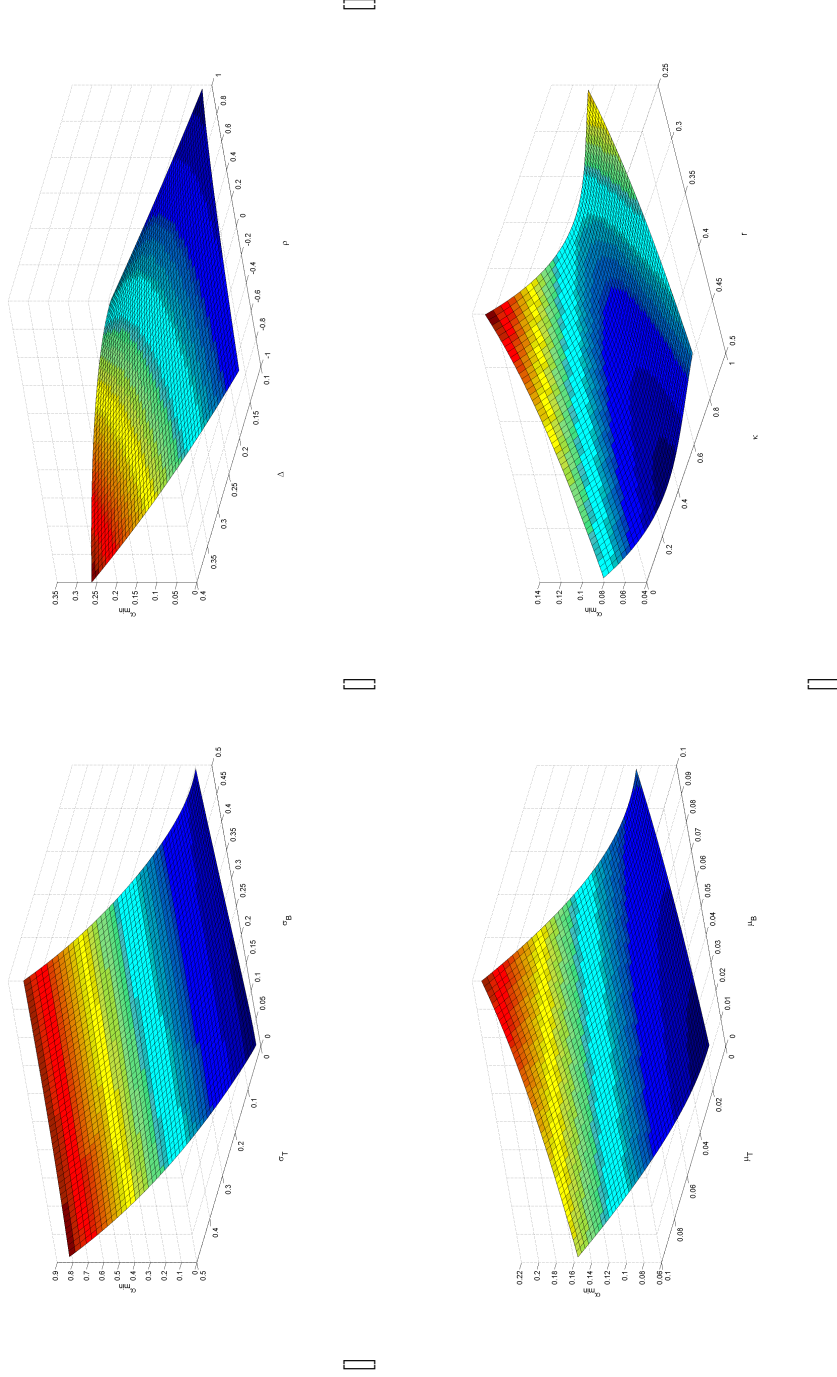
Solving  $\alpha$  for the above equation gives the synergy effect threshold in a pure cash offer. Please note that analytical solution still does not exist for the above equation and therefore numerical method is required. As both analysis requires numerical solutions, it is not possible to show the convergence when  $\kappa$  is close to zero analytically. A number of parameter constellations have shown the convergence numerically. For example, one can refer to the result in right-down sub-figure for Figure 4.1. The results for  $\kappa = 0$  show perfect convergence to the cases of  $\kappa > 0$ .

### 4.3 Synergy effect threshold

Using the numerical solution to  $\alpha_{\min}$ , it is straightforward to evaluate the synergy effect threshold for a large range of parameter constellations. As one can see from figure 4.1, the synergy effect threshold does not only depend on the characteristics of the two companies involved,  $\mu_B$ ,  $\mu_T$ ,  $\sigma_B$ ,  $\sigma_T$ ,  $\rho$  and  $\Delta$ , but also on the method of payments as represented by  $\kappa$ , the fraction of payments made in shares. The following properties are observed:

**Lemma 20.** *The partial derivatives of the synergy effect threshold  $\alpha_{\min}$  required to conduct a takeover are*

- $\frac{\partial \alpha_{\min}}{\partial \mu_B} > 0$ ,
- $\frac{\partial \alpha_{\min}}{\partial \mu_T} > 0$ , except for the condition that  $\mu_{S_B}$  has a relatively high value (the interval close to 0.1)
- $\frac{\partial \alpha_{\min}}{\partial \sigma_B} > 0$ ,



**Fig. 4.1:** Synergy effect threshold  $\alpha_{\min}$  for a takeover in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_T = 0.02$ ,  $\mu_B = 0.04$ ,  $\sigma_T = 0.1$ ,  $\sigma_B = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $r = 0.25$ ,  $\kappa = 0.25$ .

- $\frac{\partial \alpha_{\min}}{\partial \sigma_T} > 0$ ,
- $\frac{\partial \alpha_{\min}}{\partial \rho} < 0$ ,
- $\frac{\partial \alpha_{\min}}{\partial \Delta} > 0$ ,
- $\frac{\partial \alpha_{\min}}{\partial r} < 0$ ,

*One can furthermore observe that the smallest synergy effect is required for a combination of cash and shares as payment for the takeover. The result is similar to that obtained in the previous tender offer model, further proving the robustness of the model.*

Clearly the results share some features with the corresponding lemma in the previous chapter such as the cases for  $r$  and  $\rho$ . It is clear that a higher discount rate negatively affects the real options premiums for both participants of a takeover deal and therefore requires a lower total takeover threshold, i.e., a lower  $\alpha_{\min}$ . In terms of parameter  $\rho$ , companies that exhibit a high correlation will only provide very limited diversification benefits as the companies are becoming more alike. As a result, the exchanging deal for both bidder and target involves less uncertainty due to the high correlation and asks for less synergy effect to justify a deal.

Regarding other parameters, both higher  $\sigma_B$  and higher  $\sigma_T$  result in a higher  $\alpha_{\min}$ . To understand the results one should be noted that this chapter introduces the effect of financial diversification. Bidder's stochastic characteristics play a role in the stochastic characteristics of the newly merged firm, which in turn affects the uncertainty of the takeover deal. For the reasonable parameters chosen in the lemma, it is shown that when bidder firm's volatility or target firm's volatility increases, they increase the total volatility/uncertainty of the deal, which therefore make the deal require higher synergy effect to justify both bidder and target's real options.

The result suggests that a higher expected growth rate of the bidder firm increases the synergy effect threshold. The impact of the bidder firm's growth rate is twofold. First, a higher  $\mu_B$  means that what the bidder is giving up has a higher expected growth rate. Everything else being equal, it would accelerate bidder's takeover decision as waiting may deteriorate the deal. Consequently, the takeover premium the bidder requires should be lower. Second, as the newly joint firm is a combination of both assets, a higher  $\mu_B$  will lead to a higher expected growth rate for the newly created firm ( $\mu_N$ ), which represents a stronger incentive for both the bidder and the target, who will receive part of the joint firm, to delay the deal. As a result, both bidder and target will need to require a higher premium to exercise their respective options. The result suggests within a reasonable range

of parameter constellation, this effect is effectively larger than the first effect and justifies the observation that a fast-growing firm needs a higher synergy effect threshold. A similar analysis can be applied to  $\mu_T$ .

With a relatively small target the synergy effect mostly arises from the bidder, thus the target is willing to accept an offer which provides it with only a small fraction of the synergy, which the bidder is happy to offer even for small synergy effect as measured by  $\alpha$ . It explains why the small  $\Delta$  is associated with a small  $\alpha_{min}$ .

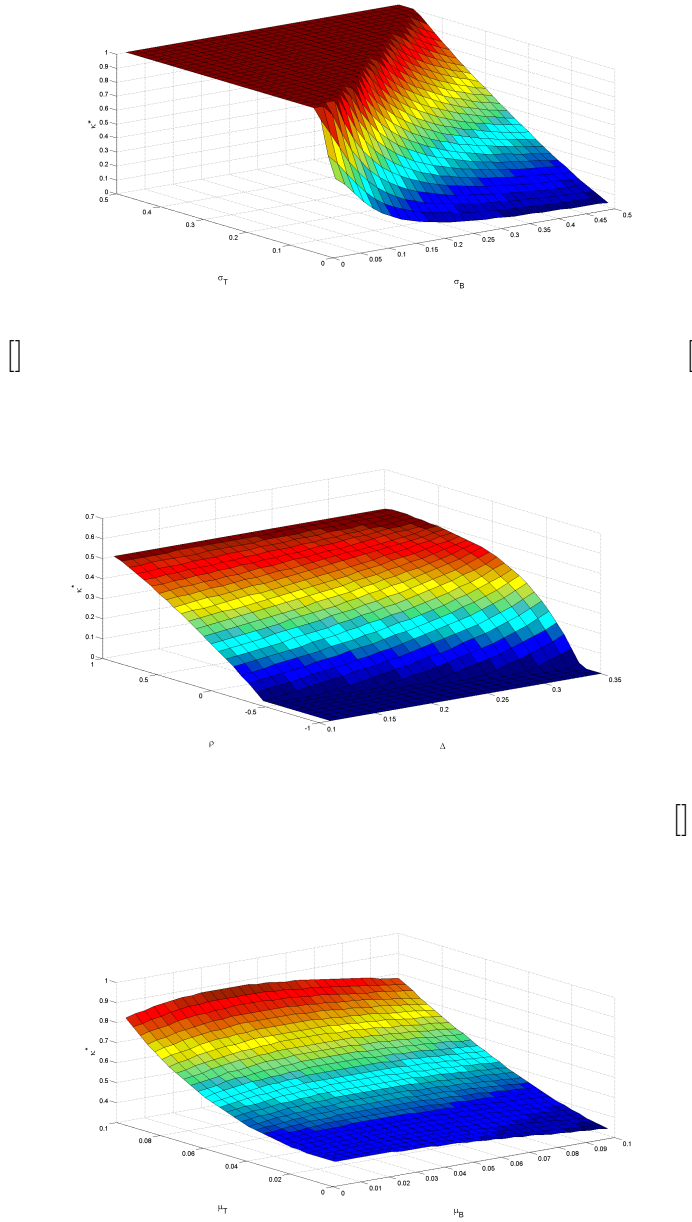
This result makes it clear that for a given synergy effect  $\alpha$  the form of payment has an impact on the decision whether a takeover goes through or not. One can thus now contemplate the medium of exchange,  $\kappa$ , which should be used to conduct the takeover. Provided companies are fully rational as assumed in this model, one would expect them to merge once the synergy effect  $\alpha$  exceeds the minimum value which is needed for any payment form  $\kappa$ , i. e. one would need to find the  $\kappa$  with the smallest  $\alpha_{min}$ . Denote the synergy effect  $\alpha_{min}^*$  and the associated payment form  $\kappa^*$ .

In figures 4.2 and 4.3 the solution to this problem for a wide range of parameter constellations is shown. Although the restriction that  $\kappa \in [0; 1]$  makes a detailed analysis more difficult once these boundaries are reached, one still can deduct a number of general properties which are summarized in the following lemma:

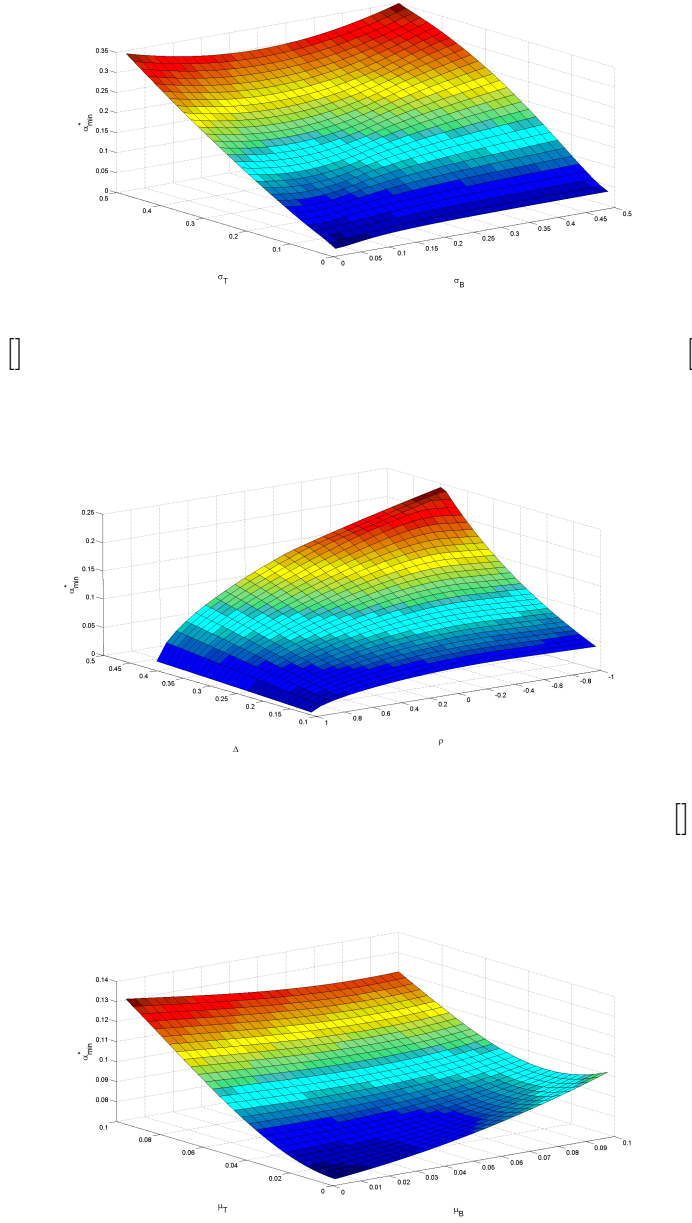
**Lemma 21.** *The partial derivatives of the fraction of shares  $\kappa^*$  which requires only the smallest synergy effects,  $\alpha_{min}^*$  are*

- $\frac{\partial \kappa^*}{\partial \mu_B} < 0$  except for the condition that  $\mu_{S_T}$  has a relatively high value (the interval close to 0.1),
- $\frac{\partial \kappa^*}{\partial \mu_T} > 0$ ,
- $\frac{\partial \kappa^*}{\partial \sigma_B} < 0$ ,
- $\frac{\partial \kappa^*}{\partial \sigma_T} > 0$ ,
- $\frac{\partial \kappa^*}{\partial \rho} > 0$ ,
- $\frac{\partial \kappa^*}{\partial \Delta} > 0$ ,

*The results on the expected return of the bidder,  $\mu_B$ , are affected by the restriction that  $\kappa \in [0; 1]$ , such that relationships are sometimes not monotonic. The above properties represent those parameter constellations which can be deemed to be the most relevant in applications.*



**Fig. 4.2:** Payment form threshold  $\kappa^*$  requiring the smallest synergy effect  $\alpha_{\min}^*$ . The base case consists of the following parameter constellation:  $\mu_T = 0.02$ ,  $\mu_B = 0.04$ ,  $\sigma_T = 0.1$ ,  $\sigma_B = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $r = 0.25$ .



**Fig. 4.3:** Smallest possible synergy effect  $\alpha_{\min}^*$  for a merger in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_T = 0.02$ ,  $\mu_B = 0.04$ ,  $\sigma_T = 0.1$ ,  $\sigma_B = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $r = 0.25$ .

It is interesting to compare the results with lemma 15 from the previous tender offer deal. One can observe the same feature for  $\rho$  and  $\mu_T$ . A higher correlation between the two firms obviously makes shares exchanging more attractive. A higher correlation reduces total uncertainty of the deal and the level of this effect should be positively related to the amount of shares employed in a deal.

The attractiveness of shares also depends on the expected growth rate of the target. The rationale has been discussed in the previous chapter. Higher proportion of shares in the payment means target firm is able to participate in the post-deal growth and therefore willing to accept lower price. This will in turn result in a lower overall synergy effect threshold.

Regarding size effect, it is found in some cases that large targets require a larger proportion of shares in the offer. One possible explanation could be that when the size of the target firm becomes closer to the bidder (i.e.,  $\Delta$  is getting close to  $1/2$ ), the financial diversification effect of the merger is greater given that correlation between two firms is low, therefore reducing the volatility of the deal. This effect should be positively related to the shares offered in the deal as target firm benefits increasingly from this effect when more shares are offered and then is able to accept lower price. It should be noted that this result is consistent with the widely discussed evidence that larger target firm increases the difficulty of raising enough cash and therefore requires more shares. However, it should also be noted that the real options model does not consider any constraints on the availability of cash - the result is simply the outcome of rational decision-making by the target and bidder.

The result on the volatility of the target firm is consistent with what is obtained in the previous chapter - higher target firm volatility favors the use of the more shares. A highly volatile firm essentially increases the business risk of the newly merged firm and from a bidder point of view, it is therefore willing to offer more shares to the target to share this risk.

Similarly one can derive the properties of the synergy effect threshold  $\alpha_{min}^*$  associated with  $\kappa^*$ . It is necessary to distinguish between the  $\alpha_{min}$  aforementioned and the  $\alpha_{min}^*$ .  $\alpha_{min}$  is the lowest value of the synergy effect to justify a takeover deal, in the context of real options analysis, given any specific payment form.  $\alpha_{min}^*$  is defined as the lowest value of the synergy effect to observe a deal, given  $\kappa = \kappa^*$ , i.e., when the payment form threshold is employed.  $\alpha_{min}^*$  is therefore the lowest  $\alpha_{min}$  provided that  $\kappa$  is taking the value at the range of  $[0, 1]$ .

Its properties (Figure 4.3) are summarized below:

**Lemma 22.** *The partial derivatives of the smallest synergy effect  $\alpha_{min}^*$  for which a takeover can be observed are*



- $\frac{\partial \alpha_{\min}^*}{\partial \mu_B} < 0$  when  $\mu_T$  is close to 0.1,
- $\frac{\partial \alpha_{\min}^*}{\partial \mu_B} > 0$  when  $\mu_T$  is close to 0,
- $\frac{\partial \alpha_{\min}^*}{\partial \mu_T} > 0$  except for the condition that  $\mu_B$  has a relatively high value (the interval close to 0.1),
- $\frac{\partial \alpha_{\min}^*}{\partial \sigma_B} > 0$  except for the condition that  $\sigma_T$  has a relatively high value (the interval close to 0.5),
- $\frac{\partial \alpha_{\min}^*}{\partial \sigma_T} > 0$ ,
- $\frac{\partial \alpha_{\min}^*}{\partial \rho} < 0$ ,
- $\frac{\partial \alpha_{\min}^*}{\partial \Delta} > 0$ ,

*The results on the expected return of the target,  $\mu_T$ , and the volatility parameter for bidder,  $\sigma_B$ , are affected by the restriction that  $\kappa \in [0; 1]$ , such that relationships are sometimes not monotonic. The above properties represent those parameter constellations which can be deemed to be the most relevant in applications.*

Please note that the characteristics of the smallest synergy effect  $\alpha_{\min}^*$  may not be exactly the same as  $\alpha_{\min}$ , described by lemma 20. The reason is that payment form threshold is chosen to obtain the lowest value of  $\alpha_{\min}$ . As argued in the previous chapter, any parameters affect both the payment form threshold and the synergy effect threshold and it is the combination of these two effects that determines the interaction laid out in the lemma. It should be noted that results and the rationale do share a lot of similarities with what is shown in Lemma 20 and therefore the interpretation for the results for  $\sigma_T$ ,  $\rho$  and  $\Delta$  can refer to lemma 20.

From numerical analysis of the results one can also easily see that for realistic parameter constellations synergy effect of approximately 10% are sufficient to enable a takeover. In that case the offer should consist of a sizeable fraction of both, cash and shares as indicated in Figure 4.2. For instance, with an increasing correlation between two businesses, an increasing number of deals should be financed by a combination of cash payment and the bidding firm shares. When two takeover participants approach a perfect correlation, which can be a good candidate to describe horizontal mergers, it can be shown from the results the deal should be paid approximately by half the cash and half the shares.

## 4.4 Division of takeover benefits

Apart from analyzing the payment form and synergy effect required for a takeover, it is furthermore of interest to analyze the size and division of the takeover benefits between the bidder and the target. The expected announcement returns of the bidder and the target should be affected by synergy effect as well as the size of the cash payments to target shareholders, which reduces the returns to bidders. It is of further interest to investigate the total return arising from a takeover deal as well as the division of takeover benefits. The returns to the target ( $R_T^N$ ), and bidder ( $R_B^N$ ), total return ( $R_N$ ) and the fraction of surplus of target ( $\psi$ ) are given by

$$(4.32) \quad R_T^N = \frac{S_T^{N*} - S_T^*}{S_T^*} = \frac{1}{\beta_T - 1},$$

$$(4.33) \quad R_B^N = \frac{S_B^{N*} - S_B^*}{S_B^*} = \frac{1}{\beta_B - 1},$$

$$(4.34) \quad R^N = \frac{S^{N*} - (S_B^* + S_T^*)}{S_B^* + S_T^*} = \frac{\kappa^* \alpha_{\min}^*}{\kappa^* + (1 - \kappa^*)(1 + \alpha_{\min}^*)\delta},$$

$$(4.35) \quad \psi = \frac{\delta S^{N*} + X - S_T^*}{S^{N*} - (S_B^* + S_T^*)} = \frac{(1 + \alpha_{\min}^*)\delta[1 - (1 - \kappa^*)\Delta] - \kappa^* \Delta}{\kappa^* \alpha_{\min}^*}.$$

In a pure cash offer:

$$(4.36) \quad R_T^N = \frac{-1}{\theta},$$

$$(4.37) \quad R_B^N = \frac{1}{\beta_B - 1},$$

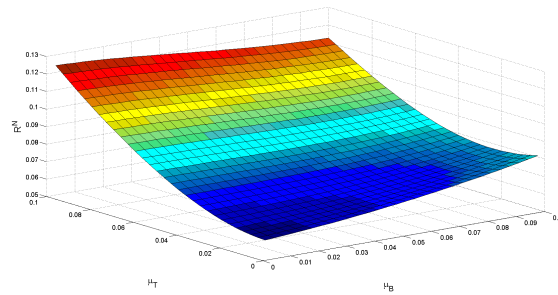
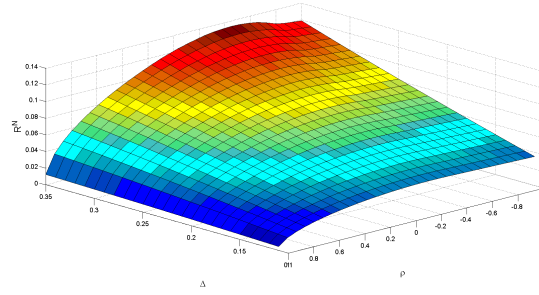
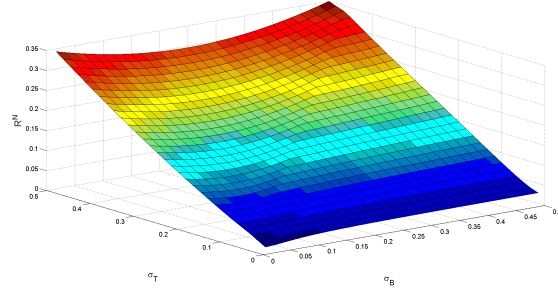
$$(4.38) \quad R^N = \alpha_{\min}^* \left( 1 - \frac{\theta - 1}{\theta} \Delta \right)$$

$$(4.39) \quad \psi = \frac{\Delta}{\alpha_{\min}^*[(\Delta - 1)\theta - \Delta]}.$$

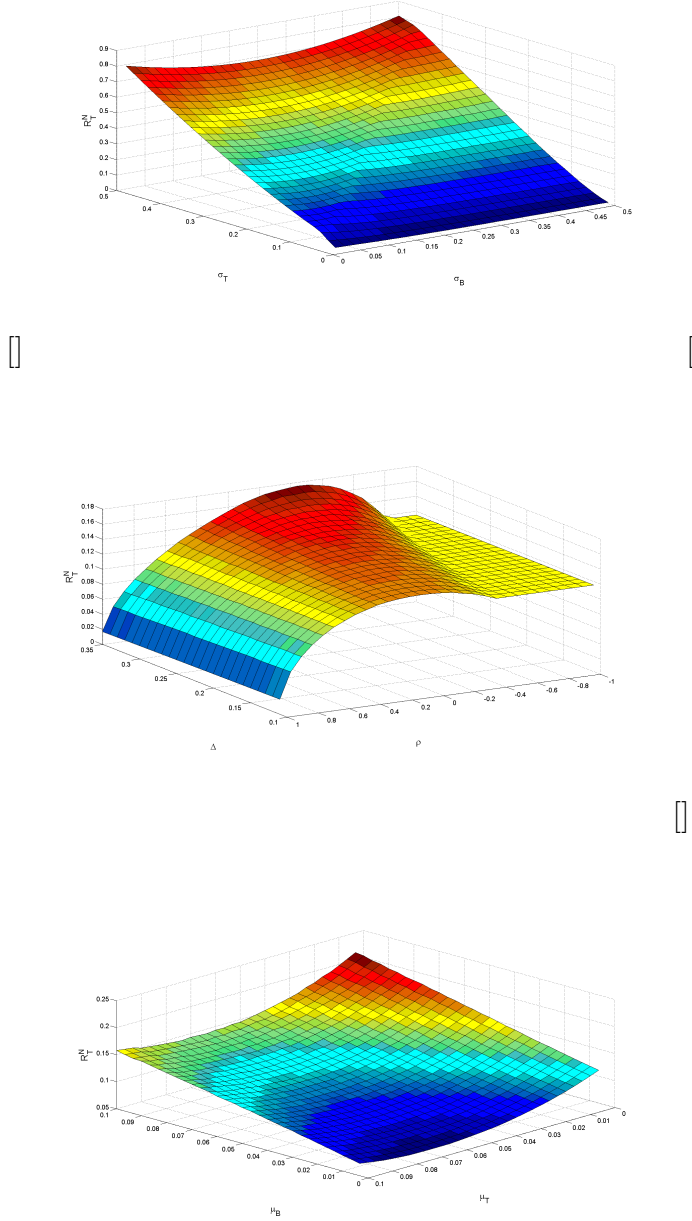
Numerical analysis is then conducted on the formulations obtained above. From figure 4.4, one can have

**Lemma 23.** *The partial derivatives of the takeover returns  $R^N$  are*

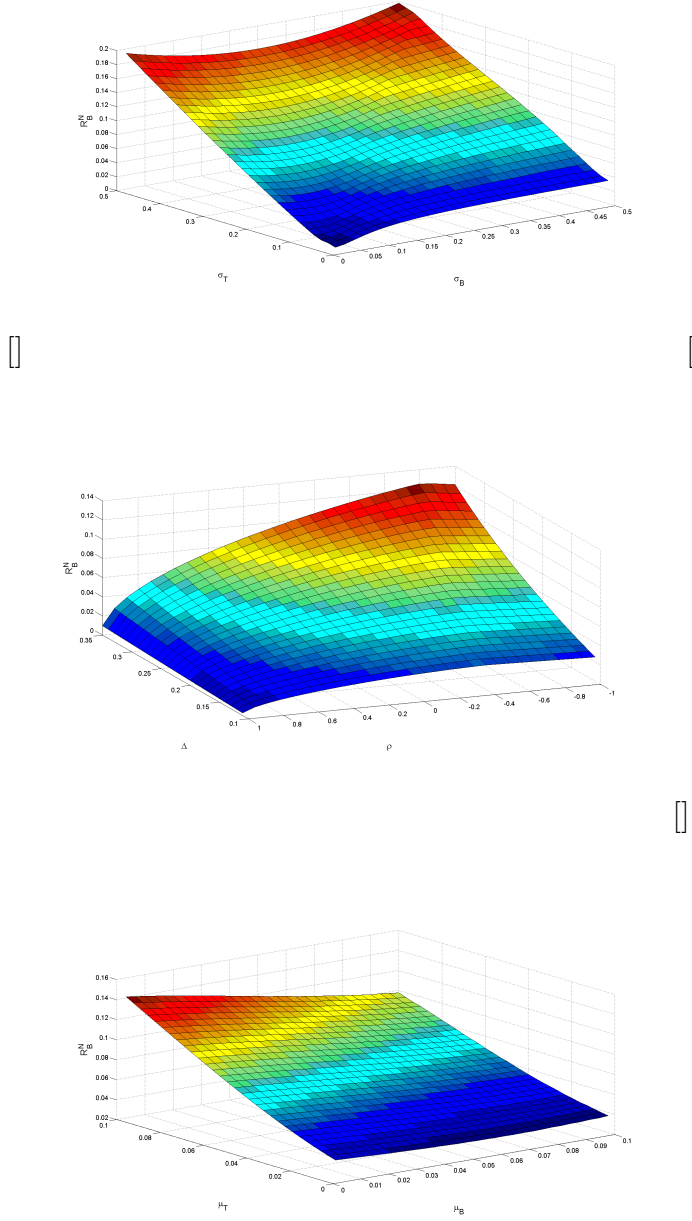
- $\frac{\partial R^N}{\partial \mu_B} < 0$  when  $\mu_T$  is close to 0.1,
- $\frac{\partial R^N}{\partial \mu_B} > 0$  when  $\mu_T$  is close to 0,
- $\frac{\partial R^N}{\partial \mu_T} > 0$  except for the condition that  $\mu_B$  has a relatively high value (the interval close to 0.1),



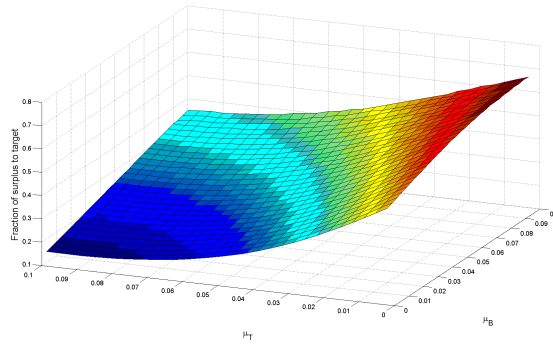
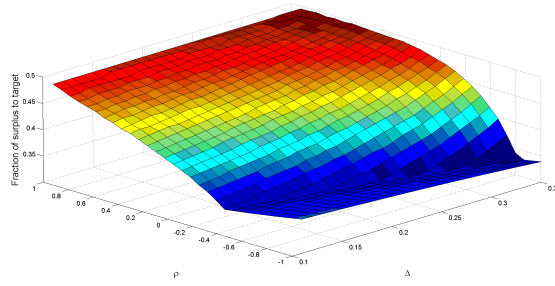
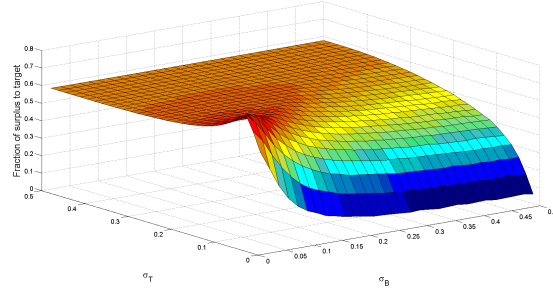
**Fig. 4.4:** Aggregate announcement returns of a takeover in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_T = 0.02$ ,  $\mu_B = 0.04$ ,  $\sigma_T = 0.1$ ,  $\sigma_B = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $r = 0.25$ .



**Fig. 4.5:** Announcement returns of a takeover to the target in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_T = 0.02$ ,  $\mu_B = 0.04$ ,  $\sigma_T = 0.1$ ,  $\sigma_B = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $r = 0.25$ .



**Fig. 4.6:** Announcement returns of a takeover to the bidder in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_T = 0.02$ ,  $\mu_B = 0.04$ ,  $\sigma_T = 0.1$ ,  $\sigma_B = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $r = 0.25$ .



**Fig. 4.7:** Fraction of merger benefits going to the takeover in dependence of a range of parameter constellations. The base case consists of the following parameter constellation:  $\mu_T = 0.02$ ,  $\mu_B = 0.04$ ,  $\sigma_T = 0.1$ ,  $\sigma_B = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $r = 0.25$ .

- $\frac{\partial R^N}{\partial \sigma_B} > 0$  except for the condition that  $\sigma_T$  has a relatively high value (the interval close to 0.5),
- $\frac{\partial R^N}{\partial \sigma_T} > 0$ ,
- $\frac{\partial R^N}{\partial \rho} < 0$ ,
- $\frac{\partial R^N}{\partial \Delta} > 0$ ,

One can easily observe that the results above are very similar to the results in lemma 22 and figure 4.3. The model assumes that the only source of the takeover returns is the synergy effect. It is therefore not difficult to understand the similarity between two lemmas. For example, a higher correlation coefficient makes two firms more alike and therefore reduces the uncertainty of the deal when more shares are used. In this case, both the synergy effect threshold and the total return of the deal should be reduced. The interpretation of other results can be done in a similar way.

Figures 4.5 and 4.6 show the respective return to bidder and target in the deal.

**Lemma 24.** *The partial derivatives of the takeover returns to target firm  $R_T^N$  are*

- $\frac{\partial R_T^N}{\partial \mu_B} > 0$ ,
- $\frac{\partial R_T^N}{\partial \sigma_T} > 0$ .

*The relationships for  $\sigma_B$ ,  $\mu_T$  and  $\rho$  are not monotonic. The size parameter  $\Delta$  hardly affects the results.*

**Lemma 25.** *The partial derivatives of the takeover returns to bidder firm  $R_B^N$  are*

- $\frac{\partial R_B^N}{\partial \mu_B} < 0$ ,
- $\frac{\partial R_B^N}{\partial \mu_T} > 0$ ,
- $\frac{\partial R_B^N}{\partial \sigma_T} > 0$ ,
- $\frac{\partial R_B^N}{\partial \rho} < 0$ ,
- $\frac{\partial R_B^N}{\partial \Delta} > 0$ ,

*The relationship for  $\sigma_B$  is not monotonic.*

The return to bidder has a very similar feature with that of the total return of the deal. It can be explained that with  $\Delta = 0.25$ , at time of the takeover, bidder firm is three times larger than target firm and therefore is expected to receive a large part of the newly created firm. Its return therefore is similar to the total synergy effect.

On the other side, the return to target firm has a greater value when the growth term of bidder firm or when target firm's volatility is higher. A higher expected growth parameter for bidder firm increases the expected growth rate of the newly created firm. This in turn increases the options value of the target firm and in turn the takeover return to the target firm. Under the conditions discussed in the numerical results, a higher volatility of target firm introduces greater deal uncertainty for the target hence it requires more return to exercise its takeover option.

Empirically it is widely accepted that the vast majority of takeover benefits goes to the target. With the assumption that the target only gets offered its reservation price (i.e., only justify its options value), it is not surprising that the bidder obtains a substantial fraction of merger benefits. Nevertheless, as figure 4.7 shows, the target can obtain a large fraction of the benefits under some conditions, e.g, when target firm has a low growth rate with high volatility. This is because the value of the option to wait for better merger conditions can be substantially higher for the target than the bidder. Results are summarized in the following lemma:

**Lemma 26.** *The partial derivatives of the fraction of takeover benefits going to target  $\psi$  are*

- $\frac{\partial \psi}{\partial \mu_B} > 0$ ,
- $\frac{\partial \psi}{\partial \mu_T} < 0$ ,
- $\frac{\partial \psi}{\partial \sigma_B} < 0$
- $\frac{\partial \psi}{\partial \sigma_T} > 0$ ,
- $\frac{\partial \psi}{\partial \rho} > 0$ ,
- $\frac{\partial \psi}{\partial \Delta} > 0$ ,

Despite that the majority of takeover benefits goes to the bidder in general, the target obtains a large return due to its relatively small size. It thus confirms the widely discussed result that returns to targets are higher. More empirical implications from the results will be summarized at the following section.



## 4.5 Empirical implications

Based on the results obtained in the previous two sections I can now collect a number of properties that allow the validity of the model to be tested. The following relationships can be tested using data from actual takeovers:

The takeover offer has an increasing fraction of share payments if the

- volatility of the bidder decreases,
- volatility of the target increases,
- target performance (expected growth rate) increases,
- correlation of the two companies increases.
- size of the target increases

The target receives a larger fraction of the takeover benefits if the

- volatility of the bidder decreases,
- volatility of the target increases,
- the correlation of the companies increases,
- the target becomes larger,
- bidder performance (expected growth rate) increases,
- target performance (expected growth rate) decreases,

These properties will be tested by using publicly available market data to verify the model in the later chapter.

## 4.6 Conclusions

In this chapter, a model is presented to consider a different structure how the takeover deal is conducted

- When two firms merge, financial synergy can be achieved,
- The cash payment can influence the stochastic property of the newly merged firm
- Operating synergy have a form of the value enhancement of the joint firm as opposed to the target firm only.

Despite a number of changes in assumptions as shown above, the results are very consistent with the previous chapter. The consistency of results can be seen as an indication of robustness of the model specification. In both models, results show that a higher correlation of two firms, a fast growing target firm and a more volatile target firm all contribute to an increasing attractiveness of shares payment in a mixed payment form. A lower volatility of bidder firm or a larger target firm leading to a larger fraction of shares is a new finding of the model in this chapter, which is the result of incorporating a diversification effect of the merged firm.

In terms of the announcement returns to the bidder and target, most results are consistent with the previous model: bidder returns are higher when the expected growth rate and volatility of the target are higher. The lower correlation between two firms seems to have the same impact. The low expected growth rate for the bidder is found to increase the announcement returns to bidders in this model and can be explained by considering the impact of the expected growth rate of bidder on the stochastic property of the newly created firm. In this model a new parameter is also introduced - the relative size of the target compared to the bidder- and it is found that a smaller target firm is likely to result in higher returns to bidders. For the target, both models support that the higher volatility of the target and the higher expected growth rate of the bidder lead to higher returns to targets. However, in this chapter, there is no strong evidence that the lower expected growth rate of target produces the lower returns to targets.

Numerical results are constrained by the parameters constellations used and therefore impossible to cover every possible situation. The empirical analysis in the next chapter is then conducted to provide further support to the results.

## **Chapter 5**

# **Method of payment: the US evidence**

### **5.1 Introduction**

Before this chapter starts, please note that this chapter is a joint work (please refer to the Disclaimer part at the beginning of the thesis for the detail) by my supervisor, Dr Andreas Krause, Dr Ahmad Ismail and myself and therefore “we” instead of “I” will be used throughout the context to reflect this arrangement.

In the previous chapters, a variety of real options models are constructed and developed with the aim to provide implications to one of the widely discussed topics in mergers and acquisitions literature: what drives the payment form in M&A deals? It has been obtained that, as shown in the previous chapters, a wide range of factors have significant impact on the choice of payment form and corresponding synergy allocation mechanism between bidders and targets. In this chapter, we collect data from US mergers and acquisitions markets during the period between 1985 and 2004 to examine these implications empirically.

### **5.2 Data description**

#### **5.2.1 Sample Selection**

The sample for our investigation is determined by identifying all mergers and acquisitions announced by US publicly listed bidders for publicly listed targets in the time period from 1 January 1985 to 22 April 2004 using the Thomson Financial SDC database. From this sample we exclude all financial institutions deals and any deals with a value below US\$ 1 million. We furthermore only consider those deals that were completed and resulted in the bidder gaining an ownership stake of at least 50%.

From this sample we only consider those deals where both the bidder and the target have share price data available in the CRSP database and accounting

information on COMPUSTAT. With these restrictions we identify 1,670 deals. In order to identify the synergy effect we search the SEC filings and media for estimated cost savings and revenue enhancements from the proposed deal and are able to identify 338 completed deals with all available information. Our final sample is further reduced to 94 due to the availability of data on corporate governance as detailed below.

### 5.2.2 Determination of explanatory variables

We determine the expected growths rate of the target and the bidder, TARG-RET and ACQ-RET, as well as the target and bidder volatility, TARG-VOL and ACQ-VOL, and the correlation between them, CORR, from daily data in the time window from 210 to 20 trading days prior to the merger announcement. We annualize our daily estimate to annual data over 252 trading days.

The amount of cash available to the bidder, ACQ-CASH, is determined as the ratio of cash plus marketable securities and the book value of assets as taken from the accounts at the end of the fiscal year prior to the merger announcement. The free cash flow of the bidder is similarly determined as the sales minus cost of goods sold, selling and general administrative expenses, taxes, change in net operating working capital and change in capital expenditures. We then divide this number by the book value of assets to obtain the variable ACQ-FCF.

We also obtain the price-earnings ratio of the target and bidder, TARG-PE and ACQ-PE, as the market price at the end of a fiscal year prior to the merger announcement divided by the earnings of that year. The leverage of the target and bidder, TARG-LEV and ACQ-LEV, is obtained as the ratio of the debt and the book value of assets at the end of the fiscal year prior to the merger announcement.

The premium paid by the bidder, PREMIUM, is determined as the ratio of the deal value and the market capitalization of the target. The deal value is the total value of the transaction as paid by the bidder and the market capitalization is the value of the target 2 months prior to the merger announcement.

The synergy, SYNERGY, is not determined that simple. We define this variable as the ratio of the synergy expected from the merger and the market capitalization of the target. Following HOUSTON AND RYNGAERT (2001) we collect information from the SEC filings (8-k filings and proxy statements) as well as press releases in the immediate aftermath of the merger announcement from Lexis-Nexis as in BERNILE (2005). We attempt to obtain information on estimated cost savings, revenue enhancements and any costs of the merger, such as fees paid to advisors.

We use as much detailed information as possible from these sources and in cases where a target date for the realization of synergy is given, we assume that in each year prior to this the gains were half that the following year. The final projected gains are then assumed to be perpetual and throughout we assume a tax rate of 36%. This methodology is identical to that used in HOUSTON AND RYNGAERT (2001) and BERNILE (2005).

We use these annual incremental gains from the merger to calculate its present value and after deducting the merger costs we obtain the size of the synergy. The discount factor to determine the present value is given by the cost of capital as determined from the CAPM. The beta is the weighted average of the beta of the bidder and the target, where the weights are the relative market capitalizations of the two companies 2 months prior to the merger announcement. The betas are estimated from daily data in the time window from 210 to 20 trading days prior to the merger announcement. As for market we use CRSP value weighted index and set a fixed risk premium of 7.5% p.a., in line with other similar investigations such as BERNILE (2005) use 8%, HOUSTON AND RYNGAERT (2001) 7 % and GILSON ET AL. (2000) 7.4 % and as for risk free rate we use the 10 year US government bond. In cases we obtain a negative beta, we set the beta equal to the average beta in our sample which is 0.86 for bidders and 0.71 for targets.

We also determine the relative size of the target in the joint company, REL-SIZE, defined as the ratio of the market capitalization of the target 2 months prior to the merger announcement and the combined market capitalizations of the target and bidder at that time.

We also use a corporate governance index for the target and bidder, TARG-GOV and ACQ-GOV. This index is based on the IRRC database and focuses on anti-takeover provisions and a higher score implies more anti-takeover measures, with a maximum score of 24. For more details of this index see GOMPERS ET AL. (2003). The inclusion of this index for targets and bidder reduces our sample size to 94 with data only available from 1990 onwards, but the importance of such measures for the success of mergers justifies their inclusion into our analysis.

Finally we also include the risk free rate, RATE, in form of the yield of the 10 year US government bond.

### 5.2.3 Determination of independent variables

We determine the fraction of shares (SAHRES) as the ratio of the value of the shares offered on the day the merger is announced and the total value of the transaction.

	Median	Standard deviation	Minimum	Maximum
<b>Dependent variables</b>				
SHARES	0.7864	0.3584	0.0000	1.0000
TARG-CAR	0.1405	0.1782	-0.2342	0.7705
ACQ-CAR	-0.0275	0.0794	-0.2362	0.1473
<b>Target characteristics</b>				
TARG-RET	-0.0255	0.5673	-0.9205	3.0082
TARG-VOL	0.4269	0.2496	0.1617	1.3676
TARG-PE	17.3551	40.7313	-256.2500	145.0893
TARG-LEV	0.6411	0.2062	0.0363	1.3849
TARG-GOV	10.0000	2.8053	3.0000	16.0000
SYNERGY	0.2635	1.2872	0.0001	11.0736
PREMIUM	1.6416	1.9470	0.8254	21.6833
<b>Bidder characteristics</b>				
ACQ-RET	-0.0276	2.5340	-0.9636	27.6041
ACQ-VOL	0.3894	0.3663	0.1427	3.3684
ACQ-PE	18.1383	239.1890	-840.6667	2462.5000
ACQ-LEV	0.6138	0.1753	0.0428	1.1246
ACQ-GOV	10.0000	2.9754	1.0000	15.0000
ACQ-CASH	0.0242	0.1370	0.0000	0.8461
ACQ-FCF	0.0810	0.0606	-0.1986	0.4424
<b>Deal characteristics</b>				
CORR	0.2613	0.2125	-0.0678	1.0000
REL-Size	0.2645	0.1604	0.0126	0.6988
RATE	0.0557	0.0075	0.0401	0.0766

**Tab. 5.1:** Descriptive statistics

In order to assess the market reaction to a merger, we determine the announcement returns (TARG-CAR and ACQ-CAR) as the cumulative abnormal returns in a time window from 5 days prior to the merger announcement to 5 days after. As for benchmark returns we use the market model calibrated in the time period of 210 to 20 days prior to the merger announcement and using a risk premium of 7.5% p.a. and the risk-free rate as the 10-year US government bond.

We show the descriptive statistics for the explanatory and dependent variables in table 5.1.

## 5.3 Hypotheses

The main part of the hypotheses to be examined in this chapter is the collection of findings and implications of previously developed models. We provide a brief summary of the factors that influence the payment form and returns to each

participant as follows. It is worth restating that the models developed in the thesis are distinct from most other empirical studies on takeover payment forms because of their pure focus on the risk-reward characteristics of both takeover participants, properly modelled by a real options analysis, and therefore taking no account of misvaluations in the market, moral hazard, asymmetric information or tax effects, the factors that most other studies normally incorporate.

### **Payment form choice hypotheses**

**Hypothesis 1** The mergers and acquisitions deals are more likely to be financed by a higher fraction of shares (SHARES) if the correlation (CORR) of the two companies are high.

**Hypothesis 2** The mergers and acquisitions deals are more likely to be financed by a higher fraction of shares (SHARES) if the target firm has a higher expected growth rate (TARG-RET)

**Hypothesis 3** The mergers and acquisitions deals are more likely to be financed by a higher fraction of shares (SHARES) if the target firm has a higher volatility (TARG-VOL)

**Hypothesis 4** The mergers and acquisitions deals are more likely to be financed by a higher fraction of shares (SHARES) if the bidding firm has a lower volatility (ACQ-VOL)

### **Returns to bidders hypotheses**

**Hypothesis 5** The announcement returns to bidding firms (ACQ-CAR) are increasing in the expected growth rate of the target firm (TARG-RET)

**Hypothesis 6** The announcement returns to bidding firms (ACQ-CAR) are increasing in the volatility of the target firm (TARG-VOL)

**Hypothesis 7** The announcement returns to bidding firms (ACQ-CAR) are decreasing in the expected growth rate of the bidding firm (ACQ-RET)

**Hypothesis 8** The announcement returns to bidding firms (ACQ-CAR) are decreasing in the correlation of the two companies (CORR)

**Hypothesis 9** The announcement returns to bidding firms (ACQ-CAR) are increasing in the size of the target firm in relation to the bidding firm (REL-SIZE)

**Returns to targets hypotheses**

**Hypothesis 10** The announcement returns to target firms (TARG-CAR) are decreasing in the expected growth rate of the target firm (TARG-RET)

**Hypothesis 11** The announcement returns to target firms (TARG-CAR) are increasing in the volatility of the target firm (TARG-VOL)

**Hypothesis 12** The announcement returns to target firms (TARG-CAR) are increasing in the expected growth rate of the bidding firm (ACQ-RET)

Apart from the implications from previous real options models, we also conduct empirical investigation on the widely discussed hypotheses in the literature, with the focus on the determination of the takeover payment forms. The hypotheses are described as follows.

**Hypothesis A** The mergers and acquisitions deals are more likely to be financed by a lower fraction of shares (SHARES) if the bidder has sufficient cash flows (ACQ-CASH, ACQ-FCF).

Following the free cash flow theory initially discussed by JENSEN (1986), the free cash flow should be paid out to finance mergers and acquisitions deals, giving rise to the fact that bidders with high level of free cash flows are more likely to launch a cash offer. MARTIN (1996) also provides support to the free cash flow hypothesis.

**Hypothesis B** The mergers and acquisitions deals are more likely to be financed by a higher fraction of shares (SHARES) if the target is relatively large compared to the bidder (REL-SIZE).

The predication can be justified by the fact that relatively large target increases the difficulty in financing with enough cash, therefore is more likely to be involved with shares.

**Hypothesis C** The mergers and acquisitions deals are more likely to be financed by a higher fraction of shares (SHARES) if the stock market performance for bidder is strong (ACQ-PE).

The strong market performance of bidding firm can be explained by two possible reasons. Firstly, booming stock market might indicate that the bidding firm has tremendous growth potential. As a result, the target firm is willing to accept share exchange as it can benefit from future growth of the newly-created firm (contingent-pricing characteristic). A high-growth firm with plenty of profitable investment opportunities would also like to finance



the deal by shares as it is normally associated with low level of free cash flows. Secondly, strong market performance might be the manifestation of market over-optimism concerning the growth potential, i.e., the bidder firm is overvalued. Asymmetric information between the bidder and the target on the value of the bidder shares allows the bidder to offer shares if they are overvalued and to offer cash if they are undervalued (HOUSTON AND RYNGAERT (2001)). In our model, we use ACQ-PE as the proxy of market performance of the bidder.

**Hypothesis D** : If the pre-deal performance of the target firm is bad (TARG-PE is low), the bidder tends not to keep the inefficient manager of the target firm, giving rise to cash financing more preferred. (ZHANG (2003))

These hypotheses can now serve as the basis for an empirical investigation, where we will also include additional variables such as the leverage effect (ACQ-LEV, TARG-LEV), anti-takeover provisions (ACQ-GOV, TARG-GOV), the synergy and the takeover premium (SYNERGY, PREMIUM). This will allow us a much more complete picture of the factors driving the choice of payment form.

## 5.4 Empirical methodology

When investigating the payment form of mergers and acquisitions, we use the fraction of shares as part of the total price offered by the bidder to the target shareholders as the dependent variable. This variable will be in the interval  $[0;1]$ , although it is can be favorable for the bidder to offer a fraction of shares above one, thus requiring an additional cash payment by target shareholders, or a negative fraction of shares, requiring target shareholders to sell the bidder their own stock. With such option not realistically available to bidders, the observed variable can thus be interpreted as censored such that for the observed fraction of shares,  $s^*$ , and the actually desired fraction of shares,  $s$ , are related as follows:

$$S^* = \begin{cases} 1 & \text{if } s > 1 \\ s & \text{if } 0 \leq s \leq 1 \\ 0 & \text{if } s < 0 \end{cases}$$

We thus have established that the fraction of shares offered is a censored variable and the appropriate econometric methodology to use to analyze such censored dependent variables is the Tobit model. We estimate a linear model of independent variables regressed against the latent variable  $s^*$  using maximum likelihood. This methodology is similar to FACCIO AND MASULIS (2005).

	Coefficient	z-statistics
CONSTANT	0.2246	0.2039
<b>Target characteristics</b>		
TARG-RET	0.4032**	2.1329
TARG-VOL	0.9308**	2.2609
log(TARG-PE)	-0.1162	-1.0922
TARG-LEV	0.8739**	2.4335
TARG-GOV	0.0110	0.3672
SYNERGY	0.0987	1.1831
PREMIUM	-0.7195***	-4.3981
<b>Bidder characteristics</b>		
ACQ-RET	0.2186	1.2906
ACQ-VOL	-0.2957	-1.2398
log(ACQ-PE)	0.5243***	3.3458
ACQ-LEV	-0.1079	-0.2935
ACQ-GOV	-0.0182	-0.7442
log(ACQ-CASH)	-0.0769	-1.5385
log(ACQ-FCF)	0.1420	1.6342
<b>Deal characteristics</b>		
CORR	0.2698	0.7336
REL-SIZE	0.6471	1.1757
RATE	-5.0782	-0.5498
$R^2$	0.4829	

**Tab. 5.2: TOBIT regression of payment form:** this table shows the maximum likelihood parameter estimates of a Tobit regression of the fraction of shares used for payment in a merger on various explanatory variables, explained in the main text. It is assumed that censoring occurs at 0 and 1. Significance of a parameter at the 10%, 5% and 1% level is indicated by \*, \*\*, and \*\*\*, respectively.

The announcement returns are investigated in a linear regression using a range of independent variables whose parameters are estimated using OLS.

## 5.5 Empirical investigation

As mentioned before, we have collected a number of explanatory variables, which are not limited to the variables discussed in the previous real options models, to test their impacts on the choice of the payment form.

We see clearly from table 5.2 that as predicted the expected growth rate of the target (TARG-RET) and its volatility (TARG-VOL) have a positive impact on the fraction of shares offered in a merger deal, which is consistent with our

hypotheses (Hypothesis 2 and 3).

In the previous real options analysis<sup>1</sup>, it is discussed that the volatility that matters (which in turn influences the value of takeover options and takeover threshold) is a function of both participants', volatility, and their correlation. Therefore the impact of the target's volatility on the payment form choice must be analyzed in this context. Given a reasonable estimate of the other parameters such as expected growth rate of both firms and correlation, we can argue that for most cases covered in our empirical observation, a higher volatility of the target firm can actually reduce the total volatility of the deal and the level of this effect is positively related to the amount of shares in the deal, which therefore favors the use of the shares.

The result shows a higher expected growth rate of the target tends to increase the use of the shares in the payment. The real options explanation has been discussed thoroughly in the Chapter 3 and therefore will be not repeated here. We attempt to provide alternative explanations here: if the expected growth of the target firm is low, it is normally the result of inefficient utilization of available assets or simply because available investment opportunities are not attractive. Despite that mergers are aimed to deal with this inefficiency, the newly merged firm will still has a relatively lower expected growth rate compared to acquiring a high growth target firm. Therefore, it is not hard to understand that the target firm would prefer receiving cash rather than the low growth merged firm shares. ZHANG (2003) (the target pre-deal performance hypothesis) also argues that if the target pre-deal performance (we use the expected growth rate of target firm before the deal to measure the target per-deal performance as opposed to the Return on Equity used by ZHANG (2003)) is bad, the bidder is more willing to eliminate the inefficient management of the target firm through a cash deal.

We further find that a lower premium (PREMIUM) paid by the bidder or a highly leveraged target (TARG-LEV) tends to increase the amount of shares offered; all other characteristics of the target are statistically insignificant.

A reason for the observation in relation to takeover premium could be that cash rich companies would be able to preempt its potential competitors by making a large premium bid. As a result, a high takeover premium is more likely to be associated with a cash payment. The effect of target firm's leverage can also be easily explained. When two firms are merged, the newly created firm will take on all the liability. A highly leveraged target firm requires more cash payment from the merged firm to repay the debt in due course - as a result, cash will be less favored to use in the takeover payment.

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<sup>1</sup> Please refer to model development in Chapter 3 and 4 for more detail.

Of the characteristics we investigate for the bidder, the only variable is statistically significant in explaining the payment form choice is ACQ-PE, which is consistent with our hypothesis C. Theories that provide support to this hypothesis can be found in the hypothesis section. It is worth mentioning that our models do not find strong evidence for free cash flows hypothesis (Hypothesis A).

We also conduct regressions using alternative specifications, most notably a linear OLS regression and an ordered logit regression by setting all cash offers as 0, mixed offers as 1 and all share offers as 2. Overall we find results that are broadly consistent with those reported here, suggesting that our results are robust to the exact specification of the model.

Apart from the valuation of the bidder, none of its properties seems to affect the decision, hence restrictions arising from the amount of borrowing or available cash seem to be as much absent as the size of the target. The takeover defences of the target are also not affecting the payment form, thus overcoming these obstacles or engaging in a friendly takeover seems to have no effect. Nevertheless, we might see an indirect influence as a merger overcoming takeover defences will generally result in a higher premium to be paid by the bidder; this premium reduces the fraction of shares used in the payment. This relationship might explain the statistical significance of the premium we find in the regression and it tallies well with the common observation that hostile takeovers, situations in which takeover defences become relevant, are more commonly paid for cash.

We can also investigate the determinants of cumulative abnormal returns for both the target and the bidder. For the bidder we find that the average return (ACQ-CAR) is slightly negative, consistent with other investigations. Using cumulative abnormal returns for 10 trading days following the merger announcement we conduct an OLS regression on the same set of explanatory variables as for the payment form and additionally include the payment form. We observe from table 5.3 that only two variables show significant parameters, the volatility of the target (TARG-VOL) and the governance of the bidder (ACQ-GOV). The former part of the statement is in line with our hypothesis 6. The acquisition of a risky target reduces abnormal returns as would be expected from normal risk-return relationship. We also find a weak impact of the quality of corporate governance of the bidder, the less good corporate governance the smaller the announcement returns. This result is intuitively clear as barriers to takeovers will often result in poor merger decisions due to hubris and empire building, thus the market will see merger announcements not that positively.

None of the other variables included in the regression is showing any statistical significance and are generally showing very small parameters. This result con-

	Coefficient	t-statistics
CONSTANT	0.0952	0.7449
<b>Target characteristics</b>		
TARG-RET	-0.0020	-0.0868
TARG-VOL	-0.1295**	-2.4103
log(TARG-PE)	0.0053	0.4218
TARG-GOV	0.0024	0.7564
SYNERGY	0.0068	0.5964
PREMIUM	0.0001	0.0171
<b>Bidder characteristics</b>		
ACQ-RET	-0.0004	-0.0712
ACQ-VOL	-0.0128	-0.3388
log(ACQ-PE)	-0.0008	-0.0656
ACQ-GOV	-0.0060*	-1.9319
log(ACQ-CASH)	-0.0031	-0.4422
log(ACQ-FCF)	0.0067	0.4571
<b>Deal characteristics</b>		
CORR	0.0199	0.4297
REL-SIZE	-0.0949	-1.5957
RATE	-0.9280	-0.7240
SHARES	-0.0163	-0.6123
$R^2$	0.2402	

**Tab. 5.3: OLS regression of announcement returns of bidders:** this table shows the parameter estimates of an OLS regression of the bidder announcement returns on various explanatory variables, explained in the main text. Significance of a parameter at the 10%, 5% and 1% level is indicated by \*, \*\*, and \*\*\*, respectively.

firms the common notion that all merger gains actually accumulate for the target shareholders rather than the bidder.

Conducting the same regression for the target company, we find a slightly different picture; several statistically significant parameters can be found, see table 5.4. We find that a well performing target (TARG-RET) shows a lower abnormal return, which is in line with our hypothesis 10. It provides further support to the robustness of our model results - real options analysis can have a supplemental role in explaining the payment form choice and returns to takeover participants. Empirical evidence also shows that a higher-growth target firm normally suffer from a merger deal as it is acquired by a relatively slow-growth bidder - negative synergy is achieved from targets' standpoint.

We observe that larger targets (REL-SIZE) attract lower abnormal returns. A common argument states that a relatively large target firm should benefit from its

	Coefficient	t-statistics
CONSTANT	0.4989**	2.2281
<b>Target characteristics</b>		
TARG-RET	-0.1040**	-2.5925
TARG-VOL	-0.0398	-0.4228
log(TARG-PE)	0.0130	0.5926
TARG-GOV	0.0093	1.6474
SYNERGY	0.0100	0.4973
PREMIUM	0.0091	1.0346
<b>Bidder characteristics</b>		
ACQ-RET	0.0112	1.2786
ACQ-VOL	-0.0989	-1.4955
log(ACQ-PE)	-0.0079	-0.3771
ACQ-GOV	-0.0042	-0.7651
log(ACQ-CASH)	-0.0020	-0.1627
log(ACQ-FCF)	0.0330	1.2815
<b>Deal characteristics</b>		
CORR	-0.0421	-0.5201
REL-SIZE	-0.3007***	-2.8880
RATE	0.8415	0.3749
SHARES	-0.2122***	-4.5524
$R^2$	0.5320	

**Tab. 5.4: OLS regression of announcement returns of targets:** this table shows the parameter estimates of an OLS regression of the target announcement returns on various explanatory variables, explained in the main text. Significance of a parameter at the 10%, 5% and 1% level is indicated by \*, \*\*, and \*\*\*, respectively.

strong bargaining power and therefore should be able to obtain higher proportion of the synergy created. Consequently, a higher announcement return should be expected. Clearly our empirical results do not support this argument, which concludes that returns to targets are lower when targets are relatively large. A possible explanation can focus on the fact that when targets are relatively large compared to bidders, the difficulty of financing the deals from the bidders' perspective is increased, and therefore the probability that bidders overpay is reduced due to this financing constraint. Holding the view that a large component of targets returns is from the overpayment from bidders, one should be able to explain the results.

Finally, we observe along with a large number of other observations that targets benefit more from cash offers than from share offers (SHARES) - in a highly competitive environment, a large amount of cash can be used to preempt or outbid other potential bidders, resulting in higher returns to targets. Asymmetric

information theory also states that target firm shareholders benefit from cash payment as they avoid receiving possibly overvalued shares. (TRAVLOS (1987), FISHMAN (1989), BERKOVITCH AND NARAYANAN (1990))

In this chapter, we find that only a small number of factors seem to influence the choice of payment form. Nonetheless, it is worth mentioning that almost all the hypotheses tested here are the implications of the models that ignore both competitive structure among bidders and asymmetric information between bidders and targets. It would also be interesting to evaluate whether other factors influence the decisions, e.g. whether they are accumulating at specific periods of time, but such an investigation is beyond the scope of the thesis and requires future effort.

## 5.6 Conclusion

This chapter conducts an empirical investigation on the predictions produced by previous real options models by analyzing the US mergers during the period of 1985-2004. Tobit regression models using maximum likelihood estimation enable us to examine the payment form choice and OLS regression models are applied to study the returns to takeover participants.

The results support the hypotheses that target firm characteristic, such as its expected growth rate and volatility, positively impacts the weight of shares used in a mixed offer as predicted. In addition, a highly leveraged target firm and/or a high takeover premium results in increasing attractiveness of shares. Our empirical analysis also provides support that shares are more likely to be used when the bidder firm has a high price-earnings ratio.

Despite these new findings, we find that there is still a significant gap in our understanding of the determinants of the payment form. In future research, it might be interesting to include additional factors that are not directly related to the target, the bidder or the deal itself, but the environment in which they are announced, e.g. the general conditions of the stock market or any herding effect arising from the conditions in other mergers.



## Chapter 6

# Conclusion

Mergers and acquisitions transactions differ significantly with respect to the payment forms. In standard terms, bidders can pay the shareholders of targets with cash - either from internal free cash flows or external debt financing - or the shares of the newly created firm.

Previous research attempts in this area argue that there are a number of factors impacting the choice of payment forms, predominantly from the perspective of asymmetric information, tax consideration, manager control, and cash availability etc. While its appearance is not as frequent as either a pure cash or a pure share offer, a mixed payment scheme, i.e., a fixed amount of cash and a fixed amount of the shares of the bidder exchanged for one share of the target, has seen its importance grow in mergers and acquisitions, particularly for large deals. In relatively large transactions, the requirement of large amount of capital makes it difficult and sometimes inadequate for a single medium of payment to satisfy the terms and conditions of both bidder and target shareholders. Takeovers therefore have seen their increasing reliance on the mixed payment form.

This thesis investigates the combination of cash and share payments in takeovers, with a strong focus on the conditions of its mix and its subsequent impact on the takeovers surplus allocation strategy. The payment form is considered in the context of real options optimization process. Conditions are obtained in terms of minimizing both bidder and target's takeover options values - so that the deals can be observed with the least obstacle. This unique feature distinguishes the thesis from most of the other models in the form that the factors notably taken into account in previous work such as information asymmetry do not play any roles in decisions with respect to the payment form. In my models, takeover participants independently consider their own takeover decisions in the context of real options analysis, considering immediate payoff from the deal and potential benefit from delaying the deal, and have come to the decision to participate only when the combination of these two values is maximized. As a result of real options optimization process for both the bidder and the target, a takeover threshold, in

the form of an additional value enhancement to the combined entity, i.e., synergy, can be obtained. I subsequently define a payment form threshold as the one that results in the lowest value of the required synergy effect enough to move a takeover deal forward. This is the main focus of the discussion of Chapter 3, following a simple attempt to describe the interaction of takeover participants in Chapter 2.

Chapter 4 sets up to consider a new takeover structure. Firstly, it is argued that financial synergy can be achieved when two firms have different growth perspectives and volatilities - the idea is similar to the diversification process in a portfolio of different assets. As a result of this diversification effect, the stochastic process of the newly created firm should be adjusted accordingly and the strategy for both takeover participants will also alter accordingly. Secondly, I suggest that the cash payment can influence the stochastic property of the new firm and in turn affect the takeover strategies of each participant. At last, instead of interpreting the synergy as value enhancement to the target firm only, it is modelled as the form of value dependent on the combined value of both the bidder and the target. While these new assumptions do not suggest invalidity of the previous chapter, they provide a further step to understanding the interaction of takeover participants.

A number of implications are obtained through the analysis mentioned above.

The factors that have a positive influence on the weight of shares component in a mixed offer include:

- A high correlation of the two companies (Chapter 3 and 4)
- A fast-growth target firm (Chapter 3 and 4)
- A high-volatility target firm (Chapter 3 and 4)
- A low-volatility bidder (Chapter 4 only)
- A larger target (Chapter 4 only)

I am also able to derive announcement returns for the target and the bidding firms and specifically I find that for bidders announcement returns are higher when

- The expected growth rate of the target is high (Chapter 3 and 4)
- The volatility of the target is high (Chapter 3 and 4)
- The expected growth rate of the bidder is low (Chapter 4 only)

- The correlation of the two companies is low (Chapter 3 and 4)
- The target firm is relative small compared to the bidder firm (Chapter 4 only)

For targets, the announcement returns are higher when

- The expected growth rate of the target is low (Chapter 3 only)
- The volatility of the target is high (Chapter 3 and 4)
- The expected growth rate of the bidder is high (Chapter 3 and 4)

As shown above, while most of the results from Chapter 3 and 4 are consistent, it is clear that changing assumptions leads to slightly different results, such as the bidder volatility factoring in the payment form, relative size effect of returns to bidders, and the impact of expected growth rate of the target on targets announcement returns. The financial diversification of the two assets in the newly merged firm should play a significant role in explaining the differences.

In Chapter 5, data has been collected from the US in the time period from 1 January 1985 to 22 April 2004 using the Thomson Financial SDC database. The final sample size is 94, mainly due to availability of data on corporate governance issues. The hypotheses generated by the previous models are tested and it is found that the target characteristics such as target firm expected growth rate and target firm volatility can significantly influence the choice of takeovers payment form. This confirms the results from the theoretical models. On the bidder side, while none of the model generated variables is found to have a significant impact on the choice of the payment form, the data shows strong evidence for the relative value theory (bidder's shares will be paid when they are considered overvalued). The premium of the deal and the leverage level of the target firm are also found to have a significant influence on takeover payment form choice.

At the very end of this thesis, I would like to discuss a bit about the limitations of our models, which I hope can shed some lights on future research in this area.

One of the major limitations is that I completely ignore the interaction between bidders and targets, unlike LAMBRECHT (2004) for example. In the assumptions, each takeover participant can make its own takeover decision - through a real options type value optimization process - on a totally isolated basis. Bidding firms make their takeover bids when the combined value of immediate payoff from the deal and their expected value from potential future movements are maximized, regardless of what target firms will react to these bids. Target firms, on the other

hand, make their takeover decisions only considering their value optimization strategy, ignoring the possibility that when a bid is rejected, a better bid might never appear in the future - it might not be worth waiting! In this context, I suggest that a stochastic bargaining game (e.g. CRIPPS (1998)) setting might be better positioned to analyze the interaction among takeover participants.

Theoretical extensions could also incorporate an agency problem when the interest of managers and shareholders are not aligned - the optimal timing of the deal might not be achieved, which in turn impacts the payment form threshold. I recommend LAMBRECHT AND MYERS (2007) as a starting point to consider such issues. One can also incorporate the costs of takeover deals, i.e., the fees paid to investment bankers, auditors, lawyers and consultants, by slightly changing the optimization strategies of both participants. The costs of the deal should play a similar role as cash payment and should change the payment form structure accordingly.

In terms of the determinants of the takeover payment form, more factors need to be considered, suggested by the empirical study at a latter part of the thesis. Factors such as asymmetric information, taxation and managerial control, if properly modelled, can greatly enhance the real options analysis by gaining more practical exposures and providing more practical implications.

Finally, it will also be interesting to consider factors such as the limits and impacts of borrowing, restrictions on available cash and regulatory constraints of various payment forms. For example, the model can be further developed to consider the situation where the debt is issued to finance the cash payment when internal cash flows are not sufficient for required cash payment. The debt issuing will change the capital structure of the bidding firm, as well as the merged firm, which in turn impacts their respective stochastic property. As a consequence, the takeover strategies of both the bidder and the target should be adjusted accordingly and the payment form threshold might need to be adapted as well.

## Appendix A

## Appendix

### A.1 Proof of Lemma 1

As Dixit and Pindyck (1994) suggest, the takeover opportunity yields no cash flows up to the time when the takeover is executed, as a result, the Bellman equations in the continuation region is:

$$(A.1) \quad rF^B dt = \varepsilon(dF^B),$$

which states that over a time interval  $dt$ , the total expected return on the takeover opportunity,  $rF^B dt$ , is equal to its expected rate of capital appreciation  $\varepsilon(dF^B)$ .  $\varepsilon(dF^B)$  can be expanded using Ito's Lemma to:

$$(A.2) \quad dF^B = F_{S_T}^B dS_T + \frac{1}{2} F_{S_T S_T}^B (dS_T)^2$$

Substituting equation (2.1) for  $dS_T$  into above expression and noting that  $\varepsilon(dz_{S_T}) = 0$  gives:

$$(A.3) \quad \varepsilon(dF^B) = \frac{1}{2} \sigma_T^2 S_T^2 F_{S_T S_T}^B dt + \mu_T S_T F_{S_T}^B dt$$

Hence the Bellman equation becomes:

$$(A.4) \quad \frac{1}{2} \sigma_T^2 S_T^2 F_{S_T S_T}^B + \mu_T S_T F_{S_T}^B - rF^B = 0$$

In addition, the takeover option value must satisfy the following boundary conditions:

$$(A.5) \quad F^B|_{S_T=S_T^*} = (1 + \alpha)S_T^* - X$$

$$(A.6) \quad F_{S_T}^B|_{S_T=S_T^*} = 1 + \alpha$$

$$(A.7) \quad \lim_{S_T \rightarrow 0} F^B = 0$$

The value-matching condition (A.5) imposes equality between the values of the takeover option and the payoff of the option upon exercise. In other words, upon exercise of the takeover option, the value of it should be equal to the payoff, or the surplus that the takeover will yield. The smooth-pasting condition (A.6) ensures that the takeover occurs along the optimal path by requiring a continuity of the slopes at the trigger threshold.

Condition (A.7) arises from the observation that if  $S_T$  goes to zero, it will stay at zero with the implication of the stochastic process for  $S_T$ , therefore the option to takeover will become worthless.

The general solution to ordinary differential equation (A.4) has the expression:

$$(A.8) \quad F_{S_T}^B = AS_T^{\beta_1} + BS_T^{\beta_2},$$

where  $A$  and  $B$  are constants that yet to be determined, and  $\beta_1$  and  $\beta_2$  are known constant whose values are given by

$$(A.9) \quad \beta_1 = \frac{1}{2} - \frac{\mu_T}{\sigma_T^2} + \sqrt{\left(\frac{\mu_T}{\sigma_T^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_T^2}} > 1$$

$$(A.10) \quad \beta_2 = \frac{1}{2} - \frac{\mu_T}{\sigma_T^2} - \sqrt{\left(\frac{\mu_T}{\sigma_T^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_T^2}} < 0$$

It is clear that if the value of target firm  $S_T$  goes to zero, the value of purchasing it will become worthless, which therefore rules out the term  $BS_T^{\beta_2}$ . The value-matching and smooth-pasting conditions then become

$$(A.11) \quad AS_T^{*\beta_1} = (1 + \alpha)S_T^* - X,$$

$$(A.12) \quad A\beta_1 S_T^{*\beta_1-1} = 1 + \alpha,$$

Solving the above equation for  $S_T^*$  and  $A$  and letting  $\beta_B$  denote  $\beta_1$  gives the lemma.

## A.2 Proof of Lemma 2

The target's threshold can be derived in a similar but not exactly same fashion. The target's takeover opportunity is analogous to American put option rather than call option, which must satisfy the following equation:

$$(A.13) \quad \frac{1}{2}\sigma_T^2 S_T^2 F_{S_T S_T}^T + \mu_T S_T F_{S_T}^T - rF^T = 0$$

The boundary conditions are given by:

$$(A.14) \quad F^T|_{S_T=S_T^*} = X - S_T^*$$

$$(A.15) \quad F_{S_T}^T|_{S_T=S_T^*} = -1$$

$$(A.16) \quad \lim_{S_T \rightarrow +\infty} F^T = 0$$

The general solution again is given by:

$$(A.17) \quad F_{S_T}^F = AS_T^{\beta_1} + BS_T^{\beta_2},$$

where  $A$  and  $B$  are constants that yet to be determined, and  $\beta_1$  and  $\beta_2$  are known constant whose values have the following expression:

$$(A.18) \quad \beta_1 = \frac{1}{2} - \frac{\mu_T}{\sigma_{S_T}^2} + \sqrt{\left(\frac{\mu_T}{\sigma_T^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_T^2}} > 1$$

$$(A.19) \quad \beta_2 = \frac{1}{2} - \frac{\mu_T}{\sigma_T^2} - \sqrt{\left(\frac{\mu_T}{\sigma_T^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_T^2}} < 0$$

It is clear that if the value of target firm  $S_T$  goes to infinity, the value of selling becomes worthless, which therefore rules out the term  $AS_T^{\beta_1}$ . The value-matching and smooth-pasting conditions then become

$$(A.20) \quad BS_T^{*\beta_2} = X - S_T^*,$$

$$(A.21) \quad B\beta_2 S_T^{*\beta_2-1} = -1,$$

Solving the above equation for  $S_T^*$  and  $B$  and letting  $\beta_T$  denote  $\beta_2$  gives the lemma.

### A.3 Proof of Lemma 4

Let  $F^B(X, S_T)$  denote the bidder's takeover option value. It should satisfy the following differential equation:

$$(A.22) \quad F_X^B \mu_X X + F_{S_T}^B \mu_{S_T} S_T + \frac{1}{2} F_{XX}^B \sigma_X^2 X^2 + \frac{1}{2} F_{S_T S_T}^B \sigma_{S_T}^2 S_T^2 + F_{X S_T}^B \rho \sigma_X \sigma_{S_T} X S_T - r F^B = 0,$$

Upon the time of exercise, the immediate payoff  $((1 + \alpha)S_T^* - X^*)$  from the option must equal the value of the option  $(F^B(X^*, S_T^*))$ . In addition, to avoid arbitrage, smooth-pasting conditions must be satisfied:

$$(A.23) \quad F^B(X^*, S_T^*) = (1 + \alpha)S_T^* - X^*,$$

$$(A.24) \quad F_{S_T^*}^B(X^*, S_T^*) = 1,$$

$$(A.25) \quad F_{X^*}^B(X^*, S_T^*) = -1.$$

An additional boundary condition is given by requiring that, as the ratio of two exchanging assets  $(X/S_T)$  is sufficiently large, the ratio of option value to  $S_T$  tends to be zero. The idea behind it is not hard to explain. If takeover cost,  $X$ , is too large compared to its payoff  $(1 + \alpha)S_T$ , bidder should find the option very unattractive because the takeover opportunity would never be profitable.

$$(A.26) \quad \lim_{(X/S_T) \rightarrow +\infty} \frac{F^B(X, S_T)}{S_T} = 0$$

Intuitively, the optimal investment decision should depend on the ratio  $X/S_T$  rather than the absolute value of either  $X$  or  $S_T$ . A new variable is created:  $R \equiv X/S_T$ . Thus

$$(A.27) \quad F^B(X, S_T) = S_T f(X/S_T) = S_T f(R),$$

where  $f$  is the function to be determined.

Successive differentiation gives:

$$(A.28) \quad F_X(X, S_T) = f_R(R),$$

$$(A.29) \quad F_{S_T}(X, S_T) = f(R) - R f_R(R),$$

$$(A.30) \quad F_{XX}(X, S_T) = f_{RR}(R)/S_T,$$

$$(A.31) \quad F_{XS_T}(X, S_T) = -R f_{RR}(R)/S_T,$$

$$(A.32) \quad F_{X S_T}(X, S_T) = R^2 f_{RR}(R)/S_T.$$

Substituting them into the partial differential equation (A.22) and the boundary conditions yields:

$$(A.33) \quad \frac{1}{2}(\sigma_X^2 + \sigma_{S_T}^2 - 2\rho\sigma_X\sigma_{S_T})R^2 f_{RR}(R) + (\mu_X - \mu_{S_T})R f_R(R) - (r - \mu_{S_T})f(R) = 0,$$

with boundary conditions:

$$(A.34) \quad f(R_B) = 1 + \alpha - R_B,$$

$$(A.35) \quad f_{R_B}(R_B) = -1,$$

$$(A.36) \quad \lim_{R \rightarrow +\infty} f(R) = 0.$$



The general solution for equation (A.33) is given by:

$$(A.37) \quad f(R) = AR^{\beta_T} + BR^{\beta_B},$$

where  $A$  and  $B$  are constants, and  $\beta_1$  and  $\beta_2$  are given by

$$(A.38) \quad \begin{aligned} \beta_T = & \frac{1}{2} - \frac{\mu_X - \mu_{S_T}}{\sigma_X^2 + \sigma_{S_T}^2 - 2\rho\sigma_X\sigma_{S_T}} \\ & + \sqrt{\left(\frac{\mu_X - \mu_{S_T}}{\sigma_X^2 + \sigma_{S_T}^2 - 2\rho\sigma_X\sigma_{S_T}} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_{S_T})}{\sigma_X^2 + \sigma_{S_T}^2 - 2\rho\sigma_X\sigma_{S_T}}} > 1 \end{aligned}$$

$$(A.39) \quad \begin{aligned} \beta_B = & \frac{1}{2} - \frac{\mu_X - \mu_{S_T}}{\sigma_X^2 + \sigma_{S_T}^2 - 2\rho\sigma_X\sigma_{S_T}} \\ & - \sqrt{\left(\frac{\mu_X - \mu_{S_T}}{\sigma_X^2 + \sigma_{S_T}^2 - 2\rho\sigma_X\sigma_{S_T}} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_{S_T})}{\sigma_X^2 + \sigma_{S_T}^2 - 2\rho\sigma_X\sigma_{S_T}}} < 0 \end{aligned}$$

Boundary condition (A.36) indicates that  $A = 0$  and reduces the two solutions for  $\beta$  to a single one:

$$(A.40) \quad f(R^*) = BR_B^{\beta_B}$$

Substituting (A.40) into (A.34) and (A.35) yields:

$$(A.41) \quad B(R_B)^{\beta_B} = 1 + \alpha - R_B$$

$$(A.42) \quad B\beta_B(R_B)^{\beta_B-1} = -1$$

Solving it for  $B$  and  $R_B$  gives the optimal investment rule:

$$(A.43) \quad R^B = \frac{\beta_B}{\beta_B - 1}(1 + \alpha).$$

Please note that  $\mu_X = \kappa\mu_{S_B}$  and  $\sigma_X = \kappa\sigma_{S_B}$ . Inserting them into the expression of  $\beta_B$  completes the proof.

## A.4 Proof of Lemma 5

The threshold of target can be driven in a similar fashion. Its takeover option should justify

$$(A.44) \quad F_X^T \mu_X X + F_{S_T}^T \mu_{S_T} S_T + \frac{1}{2} F_{XX}^T \sigma_X^2 X^2 + \frac{1}{2} F_{S_T S_T}^T \sigma_{S_T}^2 S_T^2 + F_{XS_T}^T \rho \sigma_X \sigma_{S_T} X S_T - r F^T = 0$$

Its boundard conditions are given by

$$(A.45) \quad F^T(X^*, S_T^*) = X^* - S_T^*,$$

$$(A.46) \quad F_{X^*}^T(X^*, S_T^*) = 1,$$

$$(A.47) \quad F_{S_T^*}^T(X^*, S_T^*) = -1,$$

$$(A.48) \quad \lim_{(X/S_T) \rightarrow 0} \frac{F^T(X, S_T)}{X} = 0.$$

Solving them yields the results stated in the lemma.

## A.5 Proof of Lemma 7

As aformentioned, the rationale of  $\alpha_{min}^M = \alpha_{min}^S$  when  $\kappa = 1$  is obsivous. One can just replace  $X$  wih  $S_B$  in the analysis and then is able to observe the convergence.

In a pure cash offer model, i.e.,  $\kappa = 0$ , bidder has a takeover opportunity to pay cash payment  $X = C$  to acquire the target firm  $(1 + \alpha)S_T$ . On the other hand, target firm expects to give up its own firm at the value of  $S_T$  for some amounts of cash payemnt  $X = C$ . The only uncertainty involvd is the target firm's fundamental value. This has been discussed in Chapter 2, which enables me to use the results directly below. Please note that the notations and the way of describing the takeover threshold are slightly different.

The value of takeover option for bidder if the medium of payment is pure cash is

$$(A.49) \quad F^B = [(1 + \alpha)S_T^* - X] \left( \frac{S_T}{S_T^*} \right)^{\theta_1} \quad \text{for } S_T < S_T^*,$$

$$(A.50) \quad = (1 + \alpha)S_T - X \quad \text{for } S_T \geq S_T^*.$$

The takeover threshold for bidder is

$$(A.51) \quad S_T^* = \frac{1}{1 + \alpha} \frac{\theta_1}{\theta_1 - 1} X,$$

where  $\theta_1$  has the value given by

$$(A.52) \quad \theta_1 = \frac{1}{2} - \frac{\mu_{S_T}}{\sigma_{S_T}^2} + \sqrt{\left( \frac{\mu_{S_T}}{\sigma_{S_T}^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma_{S_T}^2}} > 1.$$

The takeover threshold implies that if the value of  $S_T$  is greater than  $S_T^*$ , it is optimal for bidder to submit its takeover offer.

The value of takeover option for target if the medium of payment is pure cash is

$$(A.53) \quad F^T = [X - S_T^*] \left( \frac{S_T}{S_T^*} \right)^{\theta_2} \quad \text{for } S_T > S_T^*,$$

$$(A.54) \quad = X - S_T \quad \text{for } S_T \leq S_T^*.$$

The takeover threshold for target is

$$(A.55) \quad S_T^* = \frac{\theta_2}{\theta_2 - 1} X,$$

where  $\theta_2$  has the value given by

$$(A.56) \quad \theta_2 = \frac{1}{2} - \frac{\mu_{S_T}}{\sigma_{S_T}^2} - \sqrt{\left(\frac{\mu_{S_T}}{\sigma_{S_T}^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_{S_T}^2}} < 0.$$

The takeover threshold implies that if the value of  $S_T$  is less than  $S_T^*$ , it is optimal for target to sell its firm  $S_T$

Consider a pure cash takeover transaction. The synergy effect threshold, denoted by  $\alpha_{min}^C$ , which justifies the option value to wait for both bidder and target of the deal, is given by

$$(A.57) \quad \alpha \geq \alpha_{min}^C = \frac{\theta_1}{\theta_1 - 1} \frac{\theta_2 - 1}{\theta_2} - 1,$$

where  $\theta_{1,2}$  are aforementioned.

When  $\kappa \equiv 0$ , then

$$\begin{aligned} \beta_{T,B} &= \frac{1}{2} - \frac{\kappa\mu_{S_B} - \mu_{S_T}}{\kappa^2\sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\kappa\rho\sigma_{S_B}\sigma_{S_T}} \\ &\quad \pm \sqrt{\left(\frac{\kappa\mu_{S_B} - \mu_{S_T}}{\kappa^2\sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\kappa\rho\sigma_{S_B}\sigma_{S_T}} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_{S_T})}{\kappa^2\sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\kappa\rho\sigma_{S_B}\sigma_{S_T}}} \\ (A.58) \quad &= \frac{1}{2} + \frac{\mu_{S_T}}{\sigma_{S_T}^2} \pm \sqrt{\left(\frac{-\mu_{S_T}}{\sigma_{S_T}^2} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_{S_T})}{\sigma_{S_T}^2}} \\ &= \frac{1}{2} + \frac{\mu_{S_T}}{\sigma_{S_T}^2} \pm \sqrt{\left(\frac{\mu_{S_T}}{\sigma_{S_T}^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_{S_T}^2}} \end{aligned}$$

Then it can be approved that

$$(A.59) \quad 1 - \beta_T = \frac{1}{2} - \frac{\mu_{S_T}}{\sigma_{S_T}^2} - \sqrt{\left(\frac{\mu_{S_T}}{\sigma_{S_T}^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_{S_T}^2}} = \theta_2$$

$$(A.60) \quad 1 - \beta_B = \frac{1}{2} - \frac{\mu_{S_T}}{\sigma_{S_T}^2} + \sqrt{\left(\frac{\mu_{S_T}}{\sigma_{S_T}^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_{S_T}^2}} = \theta_1$$

As a result

$$(A.61) \quad \alpha_{min}^M = \frac{\beta_T}{\beta_T - 1} \frac{\beta_B - 1}{\beta_B} - 1 = \frac{\theta_1}{\theta_1 - 1} \frac{\theta_2 - 1}{\theta_2} - 1 = \alpha_{min}^C$$

It completes the proof.

## A.6 Proof of Lemma 8

$$(A.62) \quad \frac{\partial \alpha_{min}^M}{\partial r} = \frac{8[-2(\kappa\mu_{S_B} - \mu_{S_T})^2 + (\kappa\mu_{S_B} + \mu_{S_T} - 2r)(\sigma_{S_T}^2 + \kappa^2\sigma_{S_B}^2 - 2\kappa\rho_M\sigma_{S_B}\sigma_{S_T})]}{\sigma_M^2\sqrt{\Pi_M}[2\kappa\mu_{S_B} + 2\mu_{S_T} - 4r + \sigma_M^2(-1 + \sqrt{1 + \frac{4(\mu_{S_T} - \kappa\mu_{S_B})^2}{\sigma_M^4} - \frac{4(\kappa\mu_{S_B} + \mu_{S_T} - 2r)}{\sigma_M^2}})]^2},$$

where

$$\sigma_M \equiv \sqrt{\kappa^2\sigma_{S_B}^2 + \sigma_{S_T}^2 - 2\rho\kappa\sigma_{S_B}\sigma_{S_T}}$$

and

$$\Pi_M \equiv \left(\frac{\kappa\mu_{S_B} - \mu_{S_T}}{\sigma_M^2} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_{S_T})}{\sigma_M^2}.$$

Since  $\rho \in [-1, 1]$ , I have

$$(\kappa\sigma_{S_B} - \sigma_{S_T})^2 \leq \sigma_{S_T}^2 + \kappa^2\sigma_{S_B}^2 - 2\kappa\rho\sigma_{S_B}\sigma_{S_T} \leq (\kappa\sigma_{S_B} + \sigma_{S_T})^2,$$

which proves that the term  $\sigma_{S_T}^2 + \kappa^2\sigma_{S_B}^2 - 2\kappa\rho\sigma_{S_B}\sigma_{S_T}$  is non-negative. Moreover, the fact that  $\kappa \in [0, 1]$  and the discount rate must be higher than any drift parameter leads to  $r > \mu_{S_B} \geq \kappa\mu_{S_B}$ . The term  $(\kappa\mu_{S_B} + \mu_{S_T} - 2r)$  therefore should be negative. Any other terms in the equation is obviously positive, and I can conclude the partial derivative  $\frac{\partial \alpha_{min}^M}{\partial r}$  is negative for any  $\kappa$  value. Consequently,  $\alpha_{min}^M$  is a decreasing function of discount rate  $r$ .

## A.7 Proof of Lemma 9

$$(A.63) \quad \frac{\partial \alpha_{min}^M}{\partial \mu_{S_T}} = \frac{4[(1 + 2\sqrt{\Pi_M})\sigma_M^2 + 2\kappa\mu_{S_B} - 2\mu_{S_T}](\sigma_M^2 - 2\kappa\mu_{S_B} + 2\mu_{S_T})}{\sqrt{\Pi_M}[(-1 + 2\sqrt{\Pi_M})\sigma_M^2 + 2\kappa\mu_{S_B} - 2\mu_{S_T}]^2[(-1 + 2\sqrt{\Pi_M})\sigma_M^2 - 2\kappa\mu_{S_B} + 2\mu_{S_T}]}$$

The term  $\sqrt{\Pi_M}$  and  $[(-1 + 2\sqrt{\Pi_M})\sigma_M^2 + 2\kappa\mu_{S_B} - 2\mu_{S_T}]^2$  should always be nonnegative. The results should be easily obtained.

## A.8 Proof of Lemma 10

$$(A.64) \quad \frac{\partial \alpha_{min}^M}{\partial \mu_{S_B}} = \frac{4\kappa[(1 + 2\sqrt{\Pi_M})\sigma_M^2 - 2\kappa\mu_{S_B} + 2\mu_{S_T}](\sigma_M^2 + 2\kappa\mu_{S_B} - 2\mu_{S_T})}{\sqrt{\Pi_M}[(-1 + 2\sqrt{\Pi_M})\sigma_M^2 + 2\kappa\mu_{S_B} - 2\mu_{S_T}][(-1 + 2\sqrt{\Pi_M})\sigma_M^2 - 2\kappa\mu_{S_B} + 2\mu_{S_T}]^2},$$

The term  $\sqrt{\Pi_M}$ ,  $\kappa$  and  $[(-1 + 2\sqrt{\Pi_M})\sigma_M^2 - 2\kappa\mu_{S_B} + 2\mu_{S_T}]^2$  should always be nonnegative. The results should be easily obtained.

## A.9 Proof of Lemma 11

$$(A.65) \quad \frac{\partial \alpha_{min}^M}{\partial \sigma_{S_T}} = \frac{-32(r - \kappa\mu_{S_B})(r - \mu_{S_T})(\kappa\rho\sigma_{S_B} - \sigma_{S_T})}{\sqrt{\Pi_M}\sigma_M^2[4r + \sigma_M^2 - 2\kappa\mu_{S_B} - 2\mu_{S_T} - 2\sqrt{\Pi_M}\sigma_M^2]^2}$$

Clearly, noting the  $\kappa \in [0, 1]$ , if  $\kappa\rho\sigma_{S_B} - \sigma_{S_T}$  is positive then the partial derivative will be positive. If  $\kappa\rho\sigma_{S_B} - \sigma_{S_T}$  is negative then the partial derivative should be negative.

## A.10 Proof of Lemma 12

$$(A.66) \quad \frac{\partial \alpha_{min}^M}{\partial \sigma_{S_B}} = \frac{32\kappa(r - \kappa\mu_{S_B})(r - \mu_{S_T})(\kappa\sigma_{S_B} - \rho\sigma_{S_T})}{\sqrt{\Pi_M}\sigma_M^2[4r + \sigma_M^2 - 2\kappa\mu_{S_B} - 2\mu_{S_T} - 2\sqrt{\Pi_M}\sigma_M^2]^2}.$$

Clearly, If  $\kappa\sigma_{S_B} - \rho\sigma_{S_T} > 0$ , the synergy effect threshold obtained in a mixed offer model is a strictly increasing function of  $\sigma_{S_B}$ . Otherwise they are a strictly decreasing function of  $\sigma_{S_B}$  apart from the changing point.

## A.11 Proof of Lemma 13

$$(A.67) \quad \frac{\partial \alpha_{min}^M}{\partial \rho} = \frac{32\kappa(-r + \kappa\mu_{S_B})(r - \mu_{S_T})\sigma_{S_B}\sigma_{S_T}}{\sqrt{\Pi_M}\sigma_M^2[4r + \sigma_M^2 - 2\kappa\mu_{S_B} - 2\mu_{S_T} - 2\sqrt{\Pi_M}\sigma_M^2]^2}$$

All the terms on the denominator should be positive. It is the standard assumption in real options analysis that the expected growth rate ( $\mu_{S_B}$  or  $\mu_{S_T}$ ) should be less than the discount rate  $r$ . Noting that  $\kappa \in [0, 1]$ , it is easy to obtain that  $-r + \kappa\mu_{S_B} < 0$  and  $r - \mu_{S_T} > 0$ . Consequently, the partial derivative should be negative.

## A.12 Proof of Lemma 14

The effect of  $\kappa$  on the synergy effect threshold can be analytically obtained by differentiating it with respect to  $\alpha_{min}^M$ . The value of  $\frac{\partial \alpha_{min}^M}{\partial \kappa}$  should be given by

$$(A.68) \quad \frac{8(r - \mu_{S_T})[2\kappa\mu_{S_B}^2 + 4r\sigma_{S_B}(\kappa\sigma_{S_B} - \rho\sigma_{S_T}) + \mu_{S_B}(-2\mu_{S_T} - 3\kappa^2\sigma_{S_B}^2 + 2\kappa\rho\sigma_{S_B}\sigma_{S_T} + \sigma_{S_T}^2)]}{\sqrt{\Pi_M}\sigma_M^2[4r + \sigma_M^2 - 2\kappa\mu_{S_B} - 2\mu_{S_T} - 2\sqrt{\Pi_M}\sigma_M^2]^2}.$$

Apparently the value of denominator should be always non-negative. The term  $8(r - \mu_{S_T})$  should also be positive due to the assumption that  $r > \mu_{S_T}$ . Consequently, the sign of  $\frac{\partial \alpha_{min}^M}{\partial \kappa}$  should totally depend on the term  $2\kappa\mu_{S_B}^2 + 4r\sigma_{S_B}(\kappa\sigma_{S_B} - \rho\sigma_{S_T}) + \mu_{S_B}(-2\mu_{S_T} - 3\kappa^2\sigma_{S_B}^2 + 2\kappa\rho\sigma_{S_B}\sigma_{S_T} + \sigma_{S_T}^2)$ .

Solving the equation  $2\kappa\mu_{S_B}^2 + 4r\sigma_{S_B}(\kappa\sigma_{S_B} - \rho\sigma_{S_T}) + \mu_{S_B}(-2\mu_{S_T} - 3\kappa^2\sigma_{S_B}^2 + 2\kappa\rho\sigma_{S_B}\sigma_{S_T} + \sigma_{S_T}^2) = 0$  for  $\kappa$  and noting that the parameter of  $\kappa^2$ ,  $-3\mu_{S_B}\sigma_{S_B}^2$  is negative, one can obtain the results stated in the lemma.

Another issue that should be mentioned about is under the conditions that  $r > \mu_{S_B} > 0$ ,  $r > \mu_{S_T} > 0$ ,  $\sigma_{S_B} > 0$ ,  $\sigma_{S_T} > 0$  and  $\rho \in [-1, 1]$ , I can prove that  $\sqrt{(\mu_{S_B}^2 + 2r\sigma_{S_B}^2 + \rho\mu_{S_B}\sigma_{S_B}\sigma_{S_T})^2 + 3\mu_{S_B}\sigma_{S_B}^2[\mu_{S_B}(\sigma_{S_T}^2 - 2\mu_{S_T}) - 4r\rho\sigma_{S_B}\sigma_{S_T}]}$  is greater than  $r\sigma_{S_B}^2 - \rho\mu_{S_B}\sigma_{S_B}\sigma_{S_T}$  and therefor  $\kappa_1 > \frac{\mu_{S_B}^2 + 3r\sigma_{S_B}^2}{3\mu_{S_B}\sigma_{S_B}^2} > 1$ . It then rules out the probability that  $\kappa_1$  is less than 1 and simplifies the analysis.

### A.13 Proof of Lemma 19

Let  $F^B(S_B^N, S_B)$  denote the bidder's takeover option value. It should satisfy the following differential equation:

(A.69)

$$F_{S_B^N}^B \mu_B^N S_B^N X + F_{S_B}^B \mu_B S_B + \frac{1}{2} F_{S_B^N S_B^N}^B \sigma_B^{N^2} S_B^{N^2} + \frac{1}{2} F_{S_B S_B}^B \sigma_B^2 S_B^2 + F_{S_B^N S_B}^B \sigma_{BB} S_B^N S_T - r F^B = 0,$$

The boundary conditions are given by

$$(A.70) \quad F^B(S_B^{N*}, S_B^*) = S_B^{N*} - S_B^*,$$

$$(A.71) \quad F^B(S_B^{N*}, S_B^*)_{S_B^{N*}} = 1,$$

$$(A.72) \quad \lim_{(S_B^N/S_B) \rightarrow 0} \frac{F^B}{S_B} = 0.$$

Intuitively, the optimal investment decision should depend on the ratio  $S_B^N/S_B$  and therefore a new variable is created:  $R \equiv S_B^N/S_B$ . Thus

$$(A.73) \quad F^B(S_B^N, S_B) = S_B f(S_B^N/S_B) = S_B f(R),$$

where  $f$  is the function to determined.

Successive differentiation gives:

$$(A.74) \quad F_{S_B^N}(S_B^N, S_B) = f_R(R),$$

$$(A.75) \quad F_{S_B}(S_B^N, S_B) = f(R) - R f_R(R),$$

$$(A.76) \quad F_{S_B^N S_B^N}(S_B^N, S_B) = f_{RR}(R)/S_B,$$

$$(A.77) \quad F_{S_B^N S_B}(S_B^N, S_B) = -R f_{RR}(R)/S_B,$$

$$(A.78) \quad F_{S_B S_B}(S_B^N, S_B) = R^2 f_{RR}(R)/S_B.$$

Substituting them into the partial differential equation (A.69) and the boundary conditions yields:

$$(A.79) \quad \frac{1}{2}(\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB})R^2 f_{RR}(R) + (\mu_B^N - \mu_B)R f_R(R) - (r - \mu_B)f(R) = 0,$$

with boundary conditions:

$$(A.80) \quad f(R^*) = R^* - 1,$$

$$(A.81) \quad f_{R^*}(R^*) = 1,$$

$$(A.82) \quad \lim_{R \rightarrow 0} f(R) = 0.$$

The general solution for equation (A.79) is given by:

$$(A.83) \quad f(R) = AR^{\beta_1} + BR^{\beta_2},$$

where  $A$  and  $B$  are constants, and  $\beta_1$  and  $\beta_2$  are given by

$$(A.84) \quad \beta_1 = \frac{1}{2} - \frac{\mu_B^N - \mu_B}{\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB}} + \sqrt{\left(\frac{\mu_B^N - \mu_B}{\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB}} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_B)}{\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB}}} > 1$$

$$(A.85) \quad \beta_2 = \frac{1}{2} - \frac{\mu_B^N - \mu_B}{\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB}} - \sqrt{\left(\frac{\mu_B^N - \mu_B}{\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB}} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_B)}{\sigma_B^{N^2} + \sigma_B^2 - 2\sigma_{BB}}} < 0$$

Boundary condition (A.82) indicates that  $B = 0$  and reduces the two solutions for  $\beta$  to a single one:

$$(A.86) \quad f(R^*) = AR^{*\beta_1}$$

Substituting (A.86) into (A.80) and (A.81) yields:

$$(A.87) \quad A(R^*)^{\beta_1} = R^* - 1$$

$$(A.88) \quad A\beta_1(R^*)^{\beta_1-1} = 1$$

Solving it for  $A$  and  $R$  gives the optimal investment rule:

$$(A.89) \quad R^* = \frac{S_B^{N^*}}{S_B^*} = \frac{\beta_1}{\beta_1 - 1}$$

Noting that  $S_B^{N*} = (1-\delta)(1+\alpha)(S_B^* + S_T^* - X)$ ,  $\Delta = \frac{S_T^*}{S_B^* + S_T^*}$ ,  $X = \frac{(\kappa-1)(1+\alpha)\delta(S_B^* + S_T^*)}{\delta(\kappa-1)(1+\alpha) - \kappa}$  and  $\beta_1$  is just  $\beta_B$ , the above equation becomes

$$(A.90) \quad \frac{\kappa(1+\alpha)(1-\delta)}{[(\kappa-1)(1+\alpha)\delta - \kappa](\Delta - 1)} = \frac{\beta_B}{\beta_B - 1}$$

The proof for the target's threshold can be done in a very similar fashion by replacing  $S_B^N$  with  $S_T^N$  and  $S_B$  with  $S_T$ . The threshold for target is given by

$$(A.91) \quad R^* = \frac{S_T^{N*}}{S_T^*} = \frac{\beta_T}{\beta_T - 1}$$

Noting that  $S_T^{N*} = \delta(1+\alpha)(S_B^* + S_T^* - X) + X$ ,  $\Delta = \frac{S_T^*}{S_B^* + S_T^*}$  and  $X = \frac{(\kappa-1)(1+\alpha)\delta(S_B^* + S_T^*)}{\delta(\kappa-1)(1+\alpha) - \kappa}$  the above equation becomes

$$(A.92) \quad \frac{-(1+\alpha)\delta}{[(\kappa-1)(1+\alpha)\delta - \kappa]\Delta} = \frac{\beta_T}{\beta_T - 1}$$



## A.14 Approximation errors in Chapter 3

In chapter 3, an approximation approach has been used to provide the SDE of the takeover payment form ( $X$ ), which therefore allows for an analytical solution for the takeover options for both participants. The aim of this appendix is to explore the magnitude of the approximation errors and their subsequent impacts on the takeover options thresholds and the payment form choice solutions.

The approximation has been set up as follows.

Assume that  $\kappa$  is the fraction of the shares in the takeover payment form upon the time of the deal (therefore it is a constant not a random variable), the mixed offer, which consists of a random-value shares component and a fixed-value cash component, should approximately follow a geometric Brownian motion described by

$$(A.93) \quad dX = \mu_X X dt + \sigma_X X dZ_X,$$

where  $\mu_X = \kappa\mu_B$ ,  $\sigma_X = \kappa\sigma_B$  and  $Z_X = Z_B$ .

However, according to Ito's lemma, the takeover payment form  $X$  should follow

$$(A.94) \quad dX = \mu_B S_B dt + \sigma_B S_B dZ_B,$$

and note that since  $X = S_B + C$ ,  $X$  does not follow a geometric Brownian motion.

The question would be how well the approximation works.

The first step sets up to simulate two random processes:

1. The exact process of  $X$  ( $X_{exact}$ ), which has two components:
  - The cash component that has a constant value  $1-\kappa$  and
  - The shares component that has an initial value of  $\kappa$ , but following a standard GBM with drift and diffusion terms  $\mu_B$  and  $\sigma_B$  respectively.
2. The approximated process of  $X$  ( $X_{approx}$ ), which has an initial value of 1, but over time following a GBM with drift and diffusion terms  $\mu_X$  and  $\sigma_X$  respectively.

Simulations of  $X_{exact}$  enables me to show graphically the difference between  $X$  and a GBM. The log return of each simulated path of  $X$  has been taken and 5,000 realizations of log returns are plotted against the normal distribution in a Quantile-quantile graph, as shown in Figure A.1 and A.2.

The base case parameters are given by  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\kappa = 0.25$  (Figure A.1(a)) and the case is expanded to explore the impact of different  $\kappa$ :

- $\mu_B = 0.05, \sigma_B = 0.1, \kappa = 0.95$  (Figure A.1(b)),
- $\mu_B = 0.05, \sigma_B = 0.1, \kappa = 0.05$  (Figure A.1(c)),

different level of  $\mu_B$ :

- $\mu_B = 0, \sigma_B = 0.1, \kappa = 0.25$  (Figure A.1(d)),
- $\mu_B = 0.1, \sigma_B = 0.1, \kappa = 0.25$  (Figure A.2(a)),

and, at last, different level of  $\sigma_B$ :

- $\mu_B = 0.05, \sigma_B = 0.01, \kappa = 0.25$  (Figure A.2(b)),
- $\mu_B = 0.05, \sigma_B = 0.2, \kappa = 0.25$  (Figure A.2(c)).

From the results, it is fair to conclude that the exact process of  $X$  is very close to a GBM in most cases. The quantile of the log returns of  $X$  reasonably matches the quantile of a normal distribution in all scenarios shown in our simulations. However it is worth noting that the approximations error becomes notable when the volatility is high.

The above results only demonstrate that the approximation errors to assume that  $X$  follows a GBM are within reasonable levels. It does not provide any evidence demonstrating  $\mu_X$  and  $\sigma_X$  are as well the reasonable estimates of the drift and diffusion parameters of  $X$ .

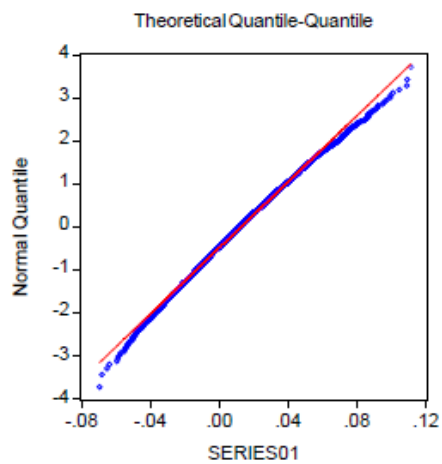
The next step therefore goes on to simulate the log returns of  $X_{approx}$  as given by equation (A.93) and  $X_{exact}$  and compare the first to fourth moment of the returns distribution in different parameter constellations.

The results are shown in Figure A.3 - 6. In each figure, approximation errors for mean returns/expected growth rate and standard deviations are calculated in relative terms. For example, given the simulated mean value of the log returns of  $X_{exact} = 0.09$  and the mean value of the log returns of  $X_{approx} = 0.11$ , the estimation error is calculated by  $\frac{2*(0.11-0.09)}{0.11+0.09} * 100\% = 20\%$ , i.e., the approximated value can be 20% different than the true value. With a different approach, the approximation errors for skewness and excess kurtosis are basically equivalent to those of  $X_{exact}$  because the log returns of  $X_{approx}$  (as a GBM) should have the skewness and excess kurtosis both equal to zero.

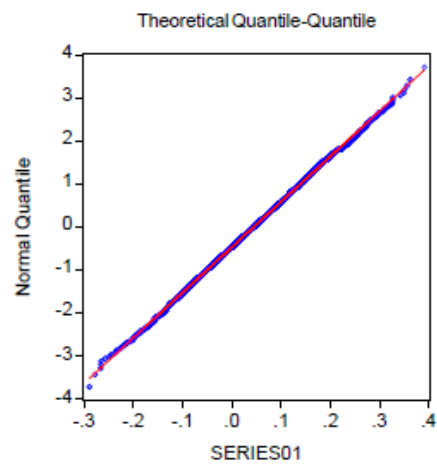
The interpretation of the results is presented below with each figure. To summarize the results, under the parameter constellation (which is  $\mu_B \in [0, 0.1]$ ,  $\sigma_B \in [0, 0.2]$  and  $\kappa \in [0, 1]$ )<sup>1</sup>:

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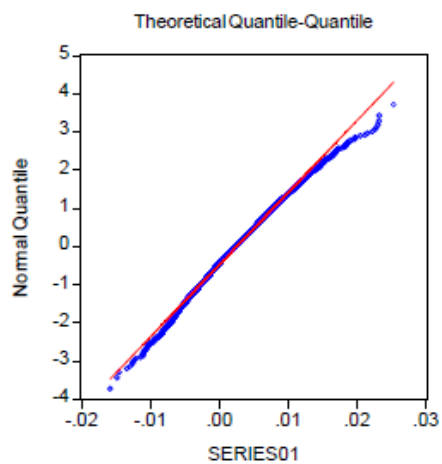
<sup>1</sup> To justify the parameter constellation I have used in the analysis, please note the  $\mu_B$  and



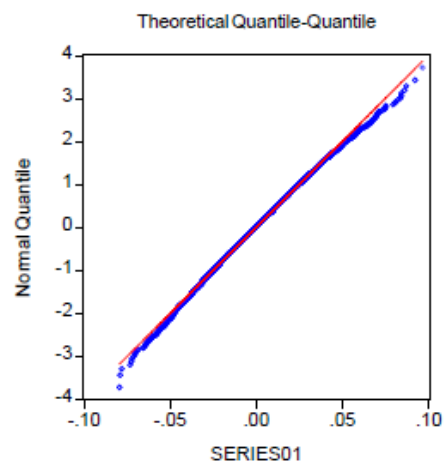
(a)  $\mu_B = 0.05, \sigma_B = 0.1, \kappa = 0.25$



(b)  $\mu_B = 0.05, \sigma_B = 0.1, \kappa = 0.95$

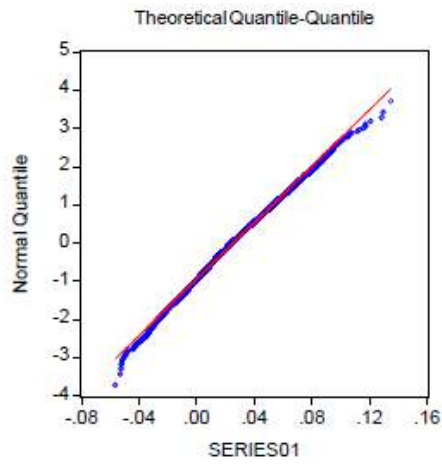


(c)  $\mu_B = 0.05, \sigma_B = 0.1, \kappa = 0.05$

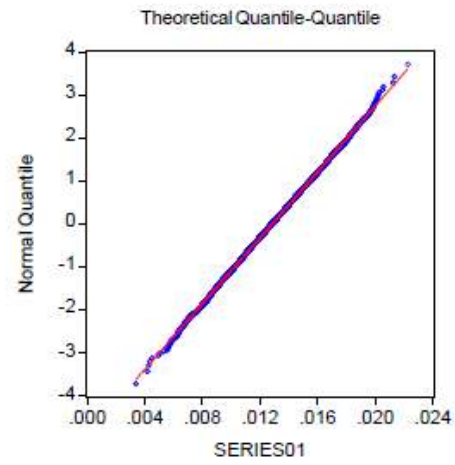


(d)  $\mu_B = 0, \sigma_B = 0.1, \kappa = 0.25$

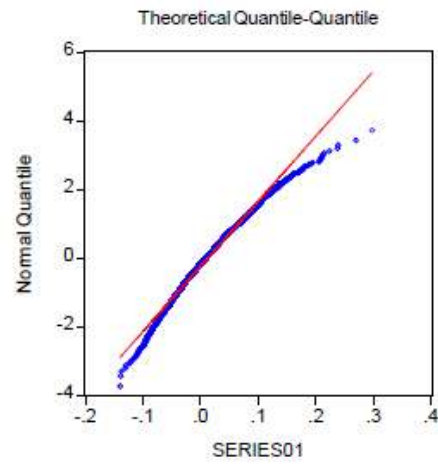
Fig. A.1: Eviews Quantile-Quantile graphs - Plot against quantiles of Normal distribution



(a)  $\mu_B = 0.1, \sigma_B = 0.1, \kappa = 0.25$

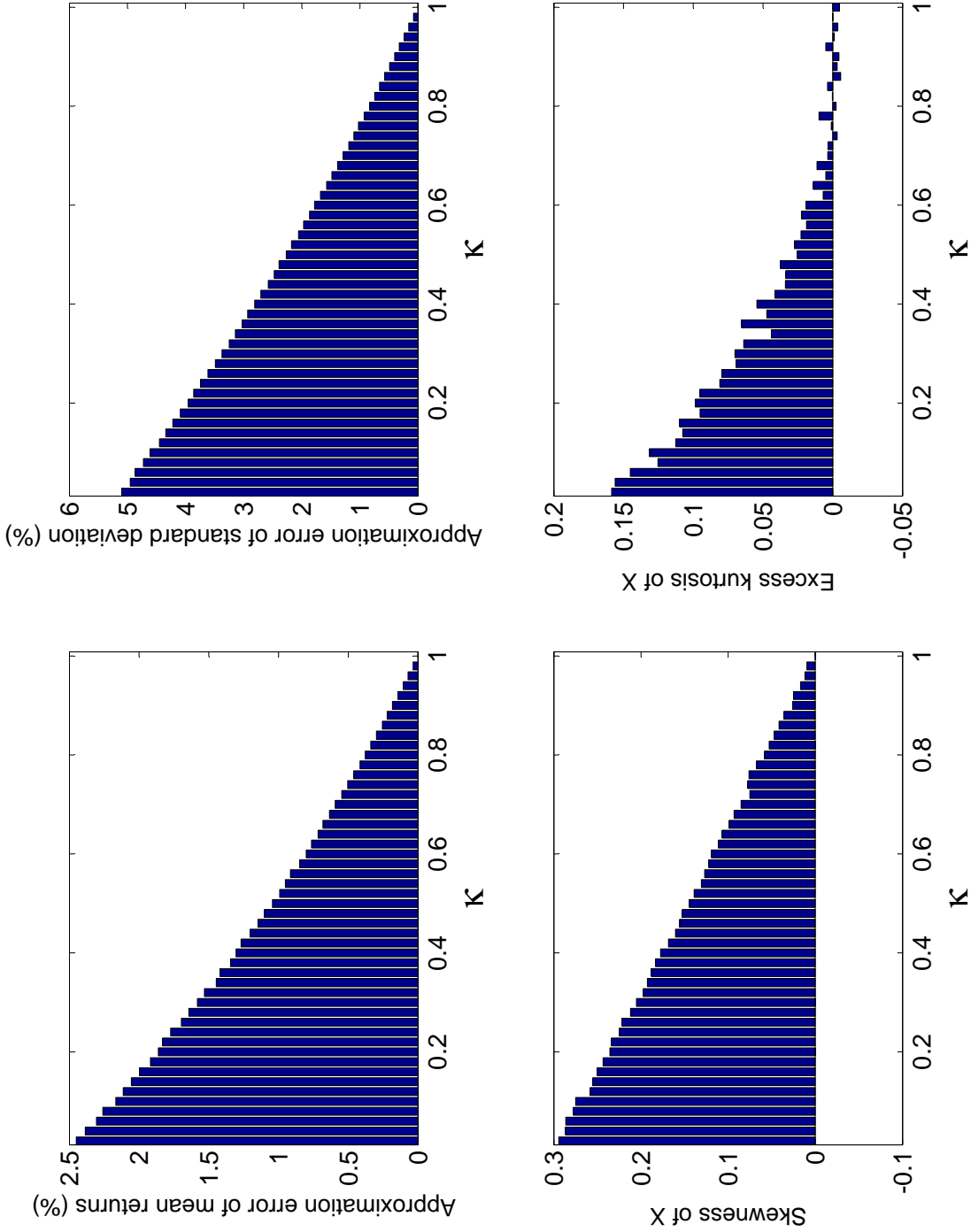


(b)  $\mu_B = 0.05, \sigma_B = 0.01, \kappa = 0.25$

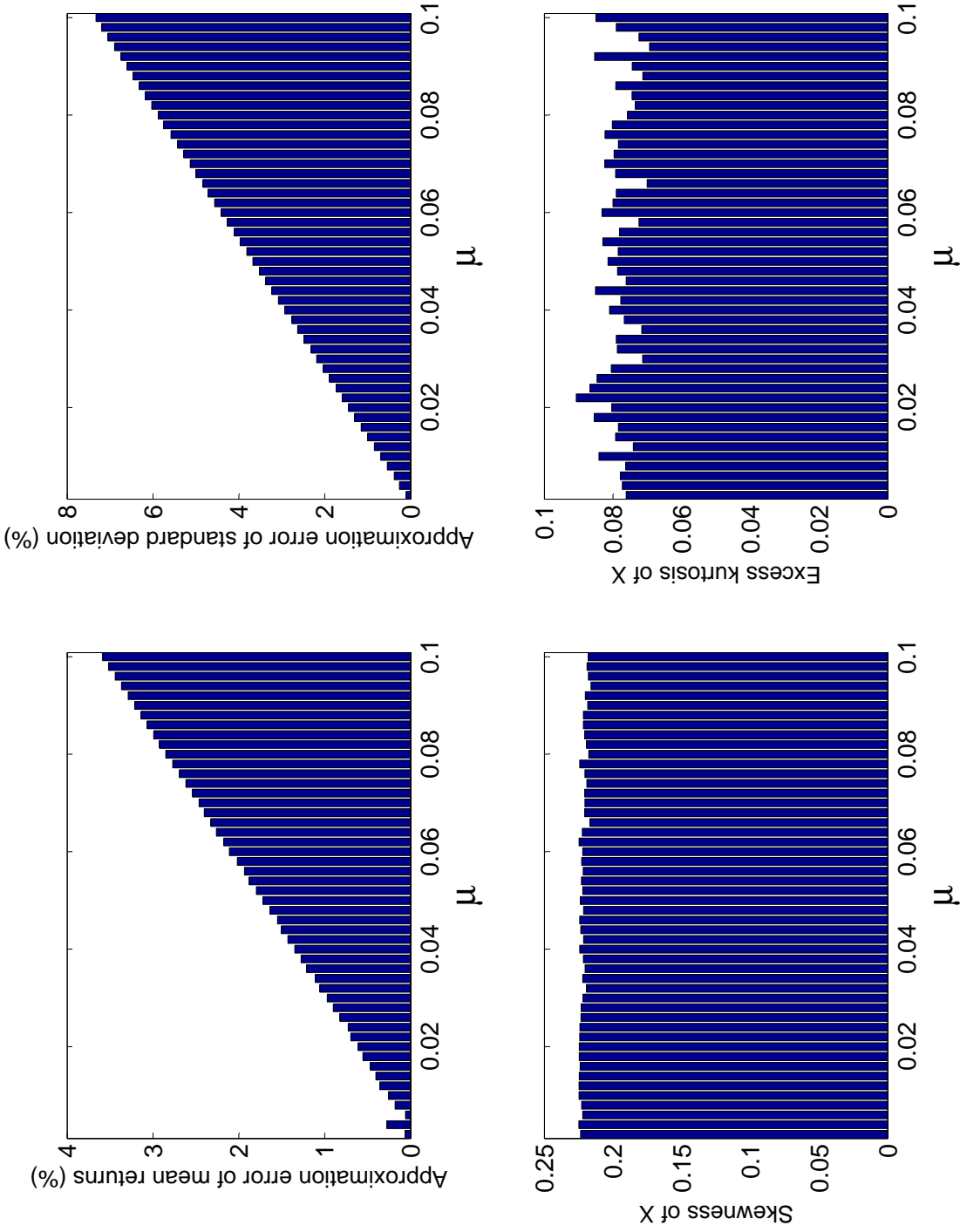


(c)  $\mu_B = 0.05, \sigma_B = 0.2, \kappa = 0.25$

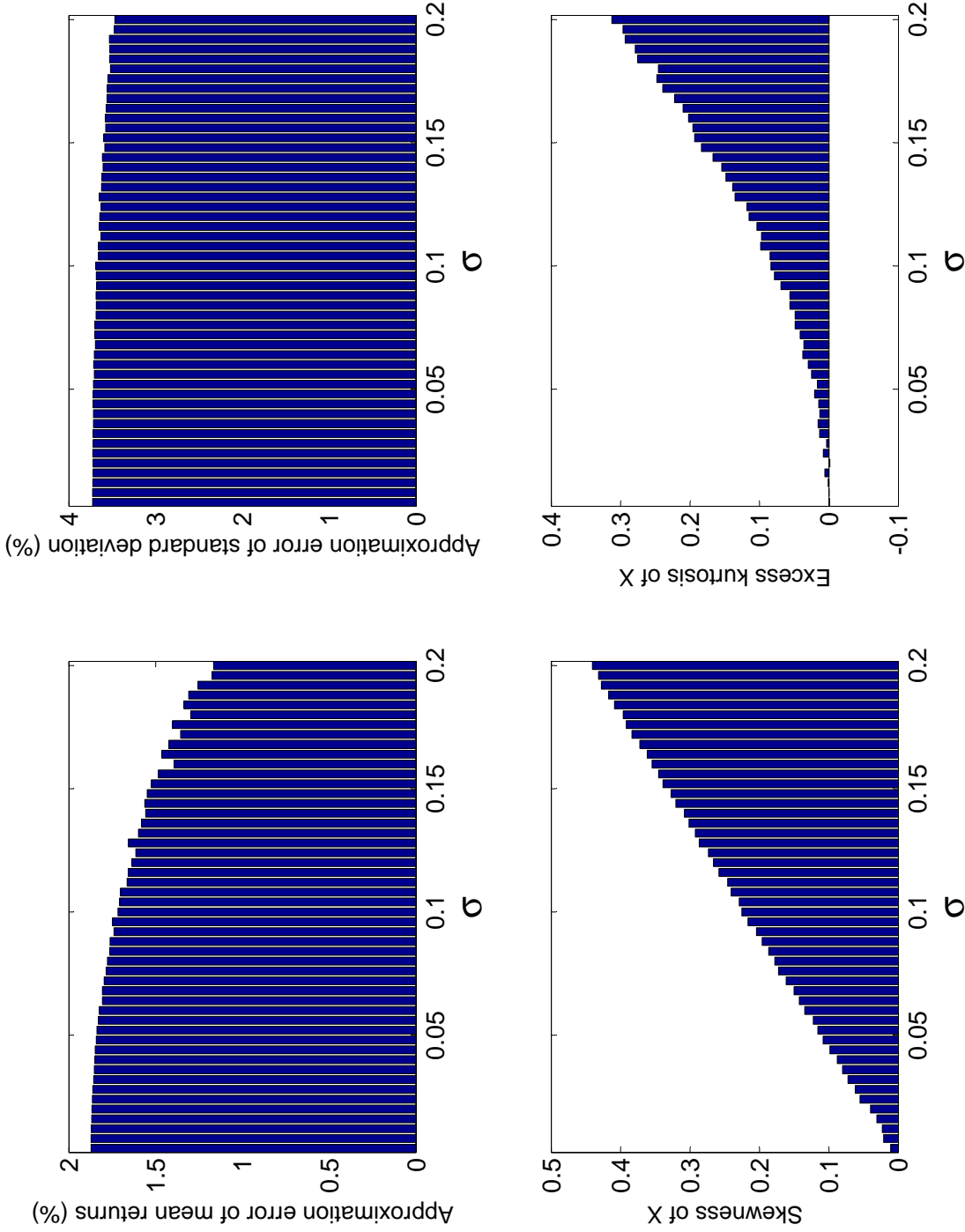
Fig. A.2: Eviews Quantile-Quantile graphs - Plot against quantiles of Normal distribution



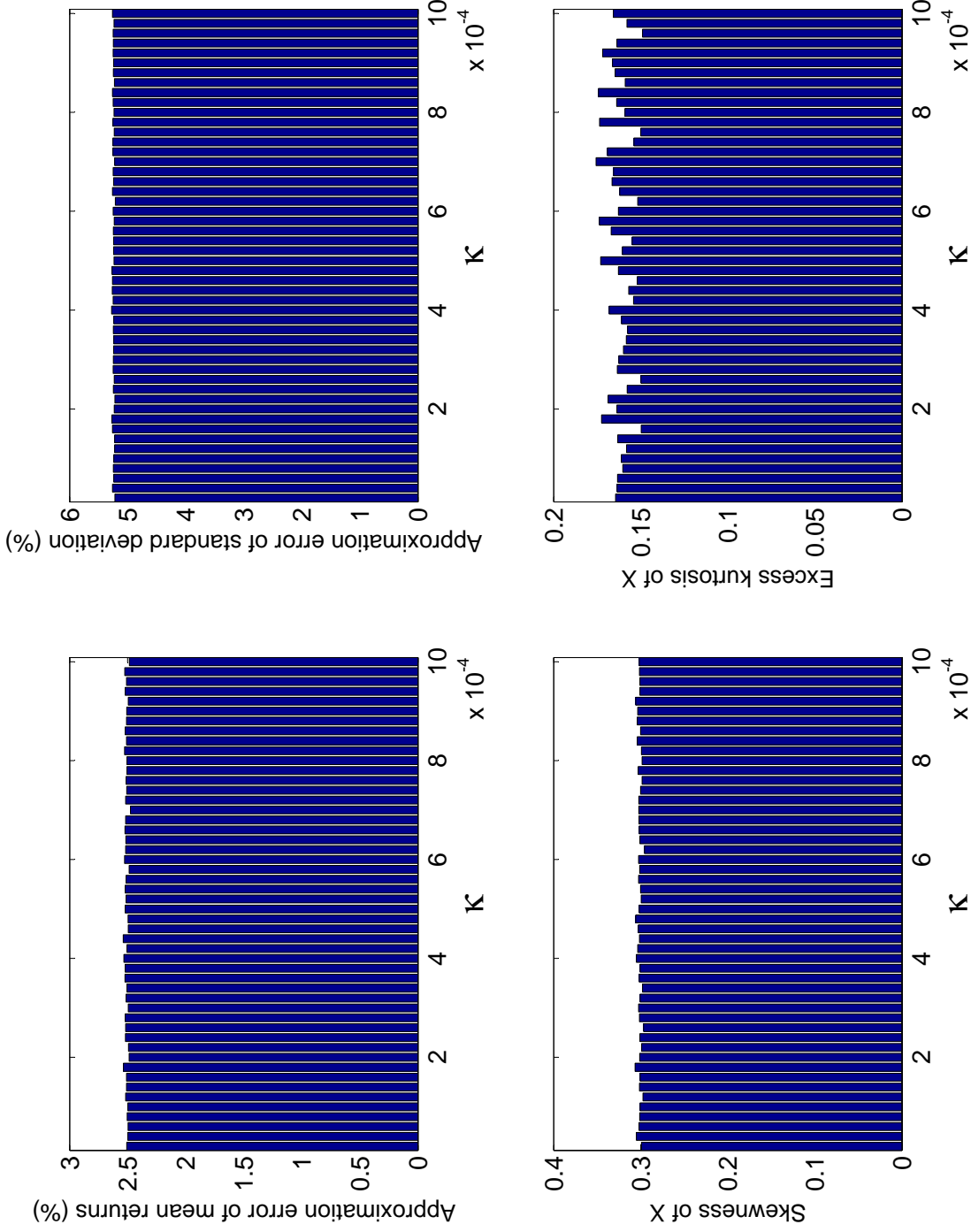
**Fig. A.3:** 20m simulations of the log returns of  $X_{exact}$  and  $X_{approx}$ . Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\kappa = 0.25$ . Approximation errors decrease for all moments when  $\kappa$  increases.



**Fig. A.4:** 20m simulations of the log returns of  $X_{exact}$  and  $X_{approx}$ . Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\kappa = 0.25$ . It is clear that approximations errors for the first two moments are positively correlated with  $\mu_B$ , while the approximations errors for third and fourth moment seem to be independent on the value of  $\mu_B$



**Fig. A.5:** 20m simulations of the log returns of  $X_{exact}$  and  $X_{approx}$ . Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\kappa = 0.25$ . The results above support the previous findings from Quantile-Quantile graphs that when volatility goes up, the skewness and the excess kurtosis become large, i.e., non-normality becomes a concern. However within our parameter constellations, the skewness and the excess kurtosis remain modest. (The highest skewness and excess kurtosis are respectively) 0.4 and 0.3 when  $\sigma_B = 0.2$



**Fig. A.6:** 20m simulations of the log returns of  $X_{exact}$  and  $X_{approx}$ . Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\kappa = 0.25$ . The motivation to check the approximation errors against very small value of  $\kappa$  ( $10^{-4}$  to  $10^{-3}$ ) comes from the concern that the idea of the approximation borrows from the literature of the spread options, where, however, the cash payment is only small amount of the overall deal. The results only show modest approximation errors even for these small value of  $\kappa$ .



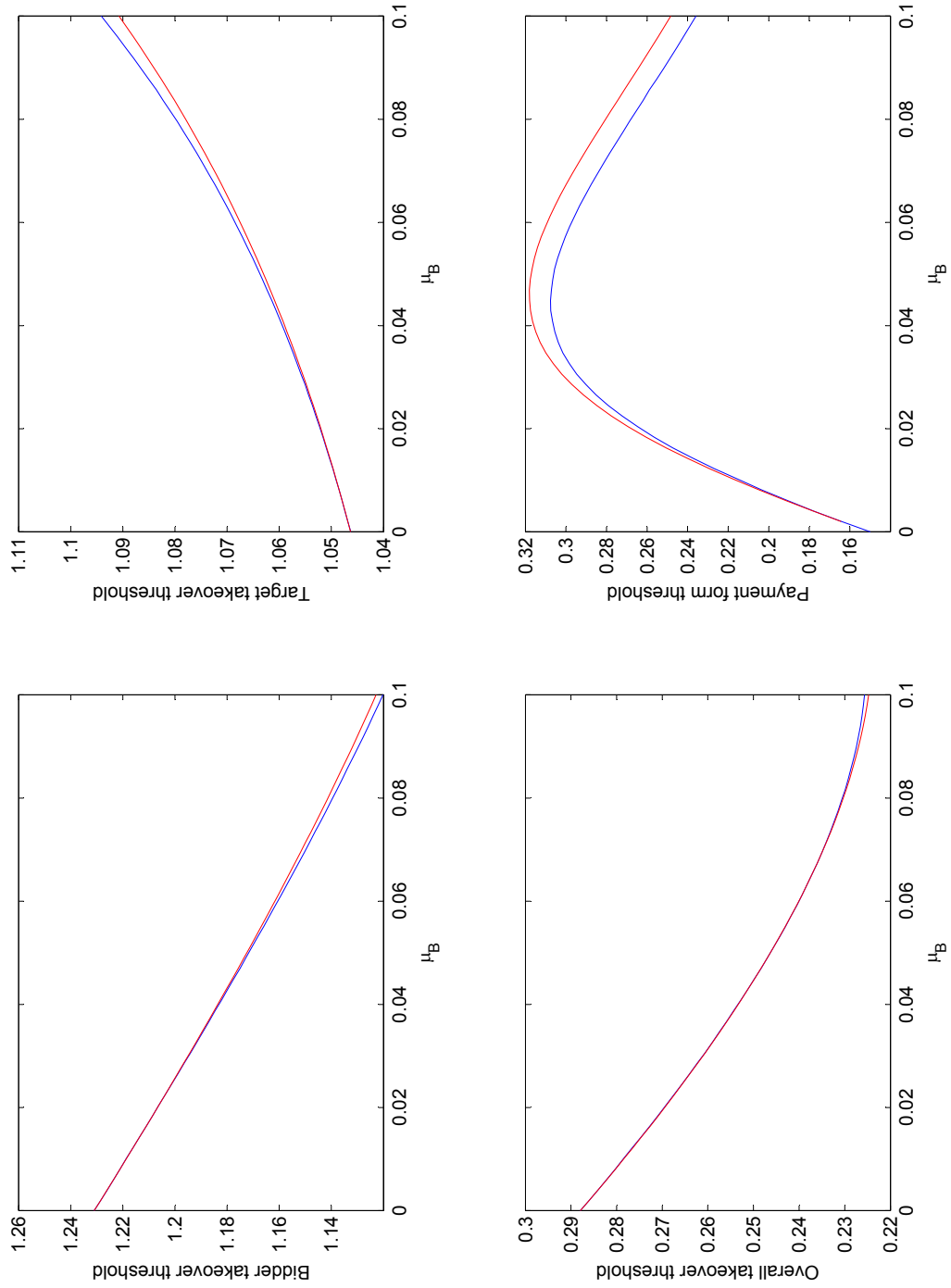
- The approximation errors for the mean are within the range of  $[0, 3.5\%]$  of the exact value,
- The approximation errors for the standard deviation are within the range of  $[0, 7\%]$  of the exact value,
- The approximation errors for the skewness are within the range of  $[0, 0.4]$ ,
- The approximation errors for the excess kurtosis are within the range of  $[0, 0.3]$ .

The last step to check the approximation errors involves using the values of  $\mu_B$  and  $\sigma_B$  from the simulated process of  $X_{exact}$  to replace the ones from  $X_{approx}$  to estimate the impact of these errors on the output of the models in Chapter 3, including the takeover threshold for each participant, the overall takeover threshold and the payment form threshold. Please note that these impacts are not exactly the errors of the approximation as essentially the approach discussed above sets up to check the errors generated from using a GBM with biased drift and diffusion terms, while in fact  $X$  does not follow exactly a GBM. Therefore, while, as demonstrated by the results above, the ignorance of non-normality issue should not lead to excess biases, I would like to remind the readers the existence of this bias.

The results are shown by Figure A.7 - 18. An interpretation of the process and the results is given below. Figure A.7 - 9 illustrates graphically the difference with regard to the takeover and payment form thresholds when the parameters from  $X_{exact}$  and  $X_{approx}$  are used respectively. Then Figure A.10 - 18 explore the magnitude of these difference. In each figure, the top-left part focuses on the approximation errors for bidder's takeover threshold given by equation (3.9). The top-right part explores the approximation errors for the takeover options of the target firm, given by equation (3.13). The bottom-left part demonstrates the approximation errors for the overall takeover threshold, i.e., the lowest required synergy effect given by equation (3.15). The above three error terms are shown in relative terms. The last part, the bottom-right part, looks at the impact of the approximation errors of the  $X$  on the payment form threshold, of which the

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$\sigma_B$  refer to the parameters of the fundamental values of the bidder firm, i.e., the present value of the future cash flows generated from its core business. If a certain degree of market inefficiency is allowed, i.e., the price can overshoot or undershoot, it is therefore reasonable to believe that the true volatility should be lower than the one implied by the stock market and 20% annualized volatility can be a reasonable cap for many businesses. A long term annual growth rate of 10%, which is multiple time of the real GDP growth in most developed countries, should be a reasonable cap of the expected growth rate for most firms.



**Fig. A.7:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$

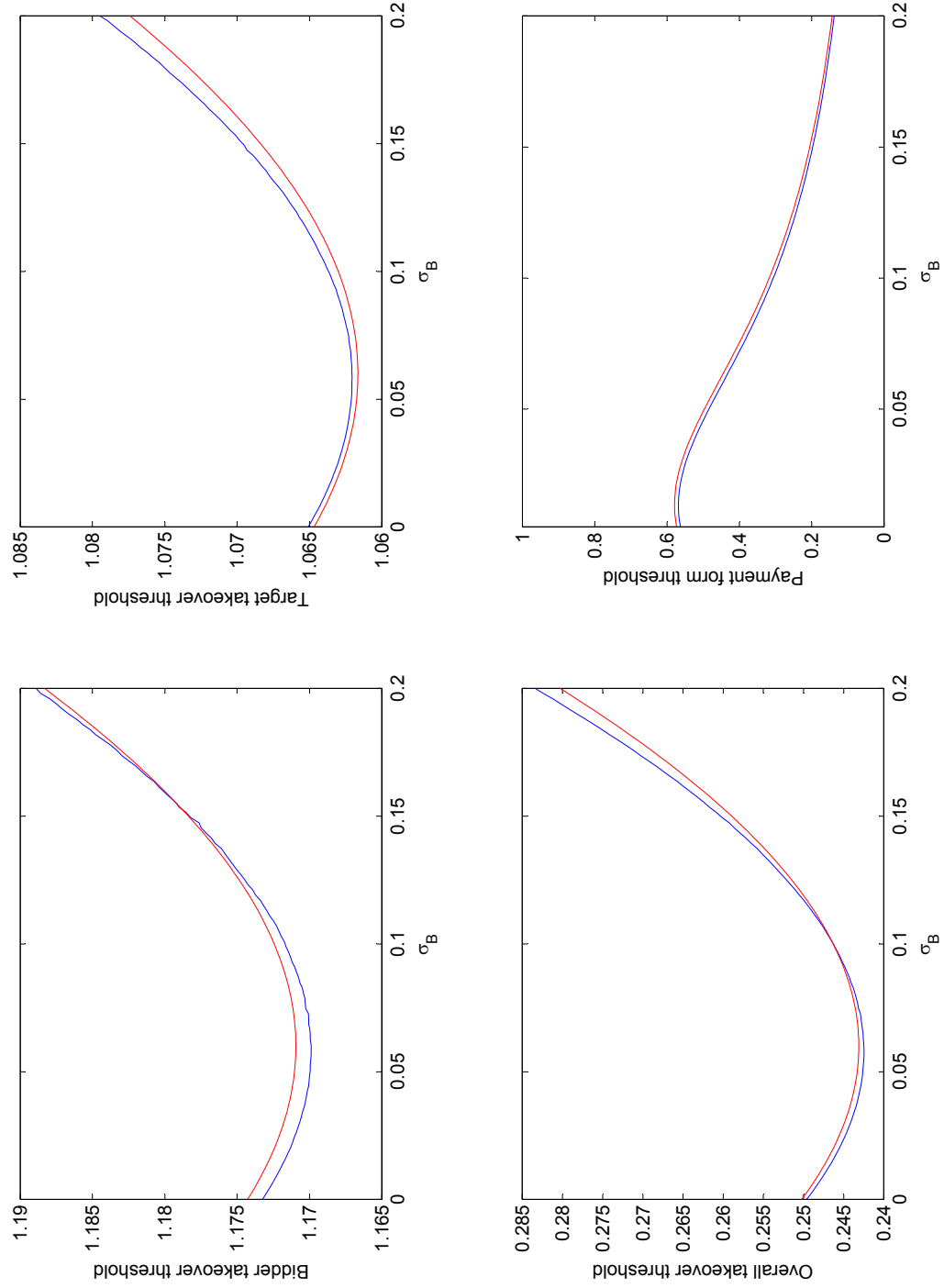
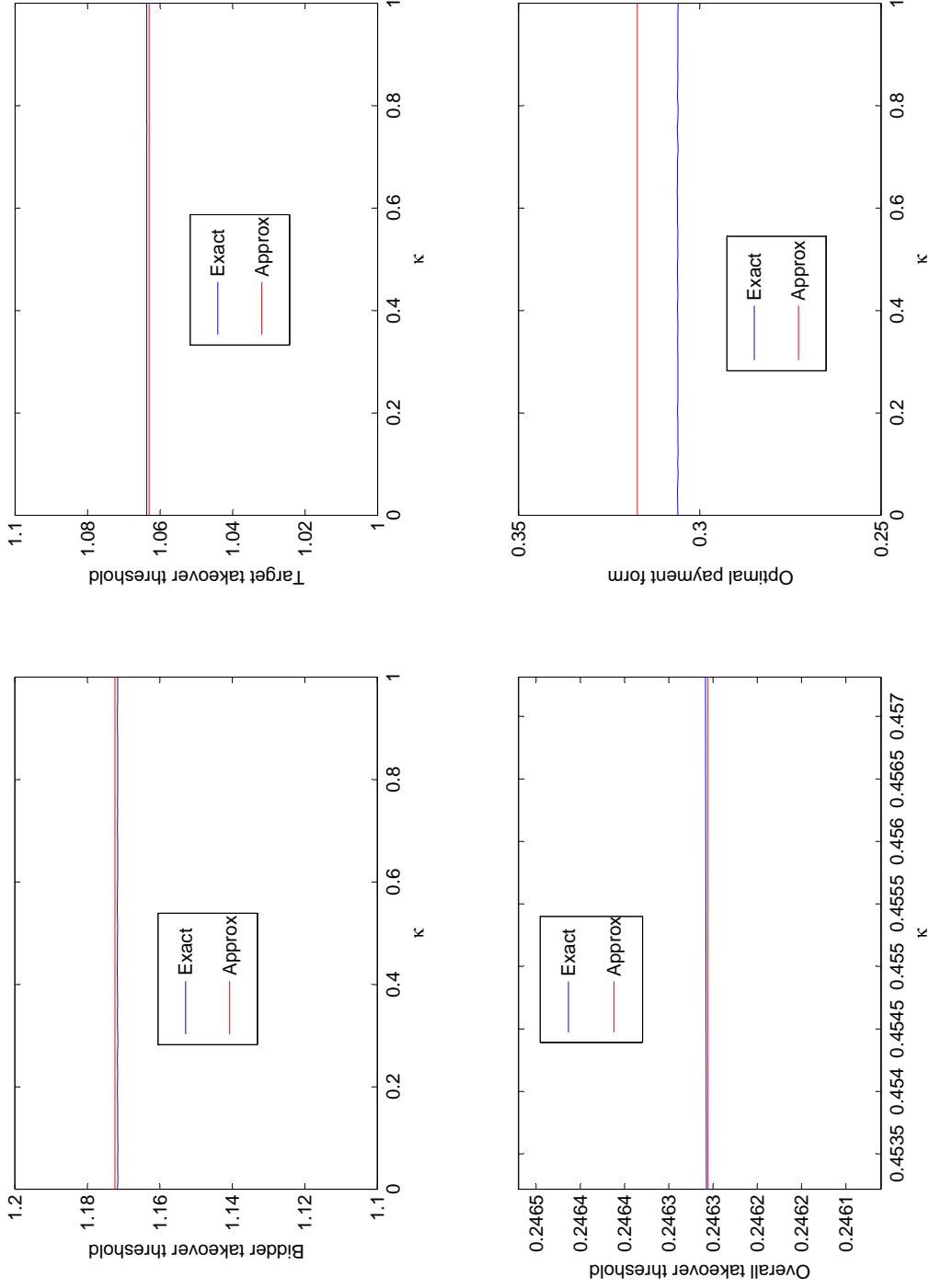
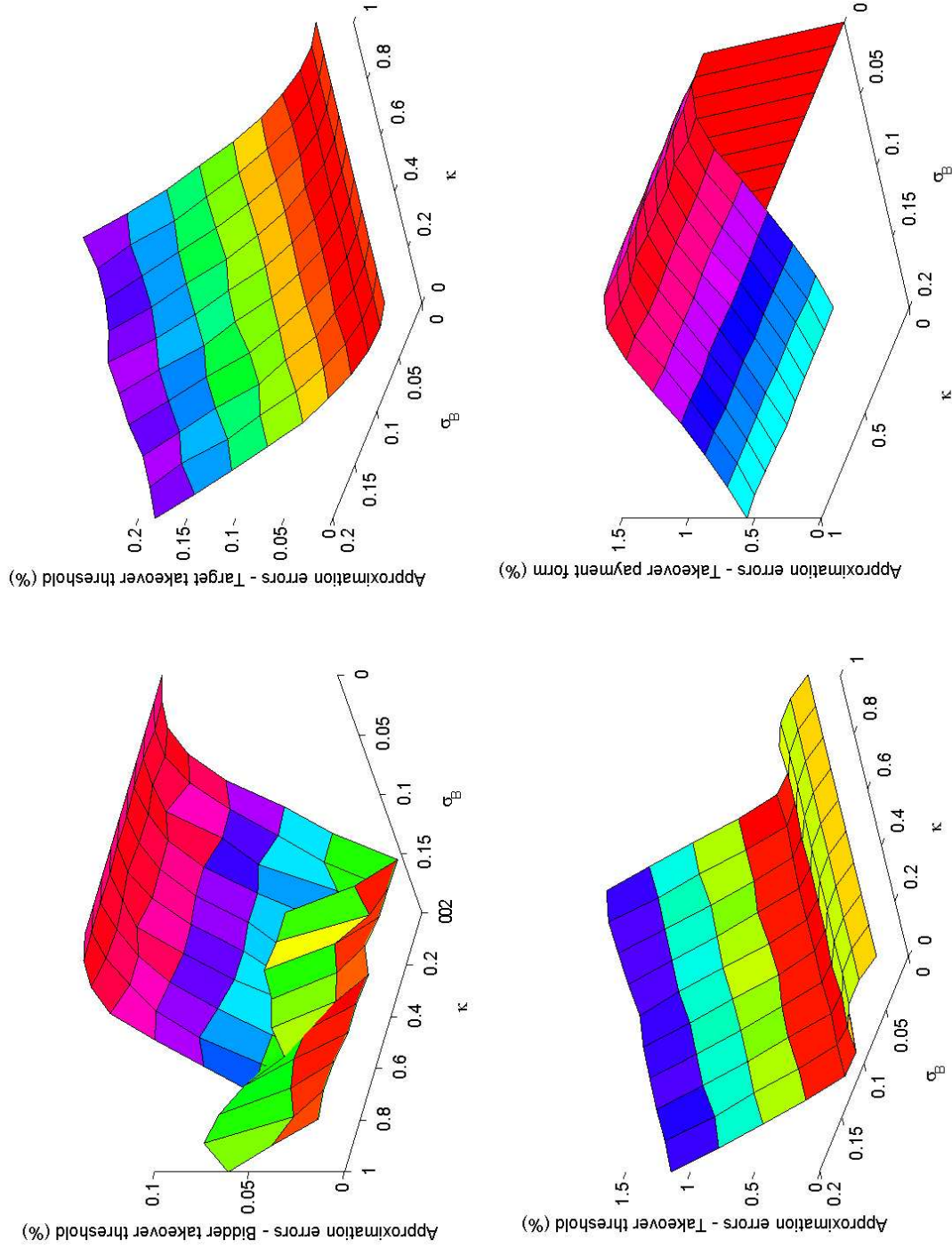


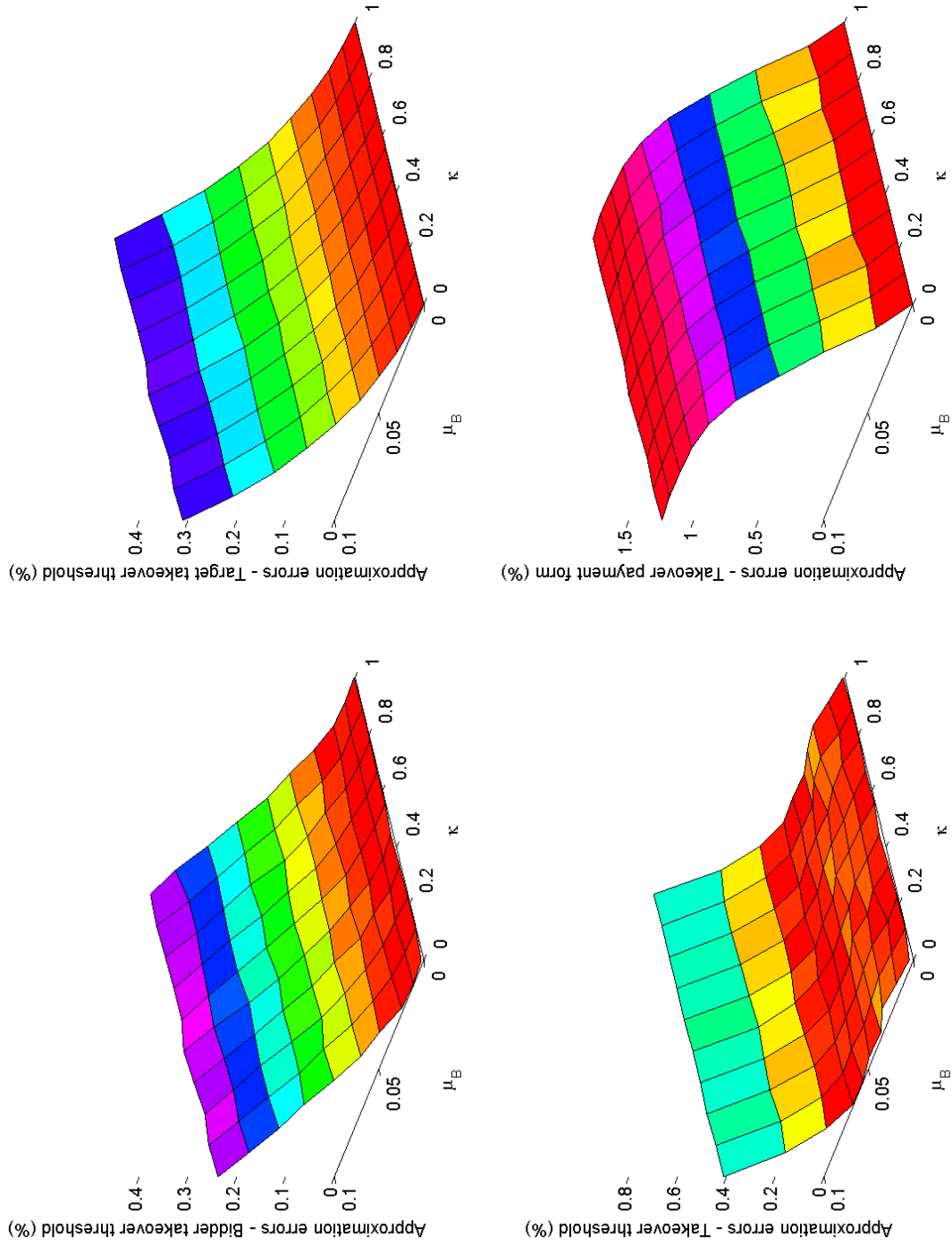
Fig. A.8: Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$



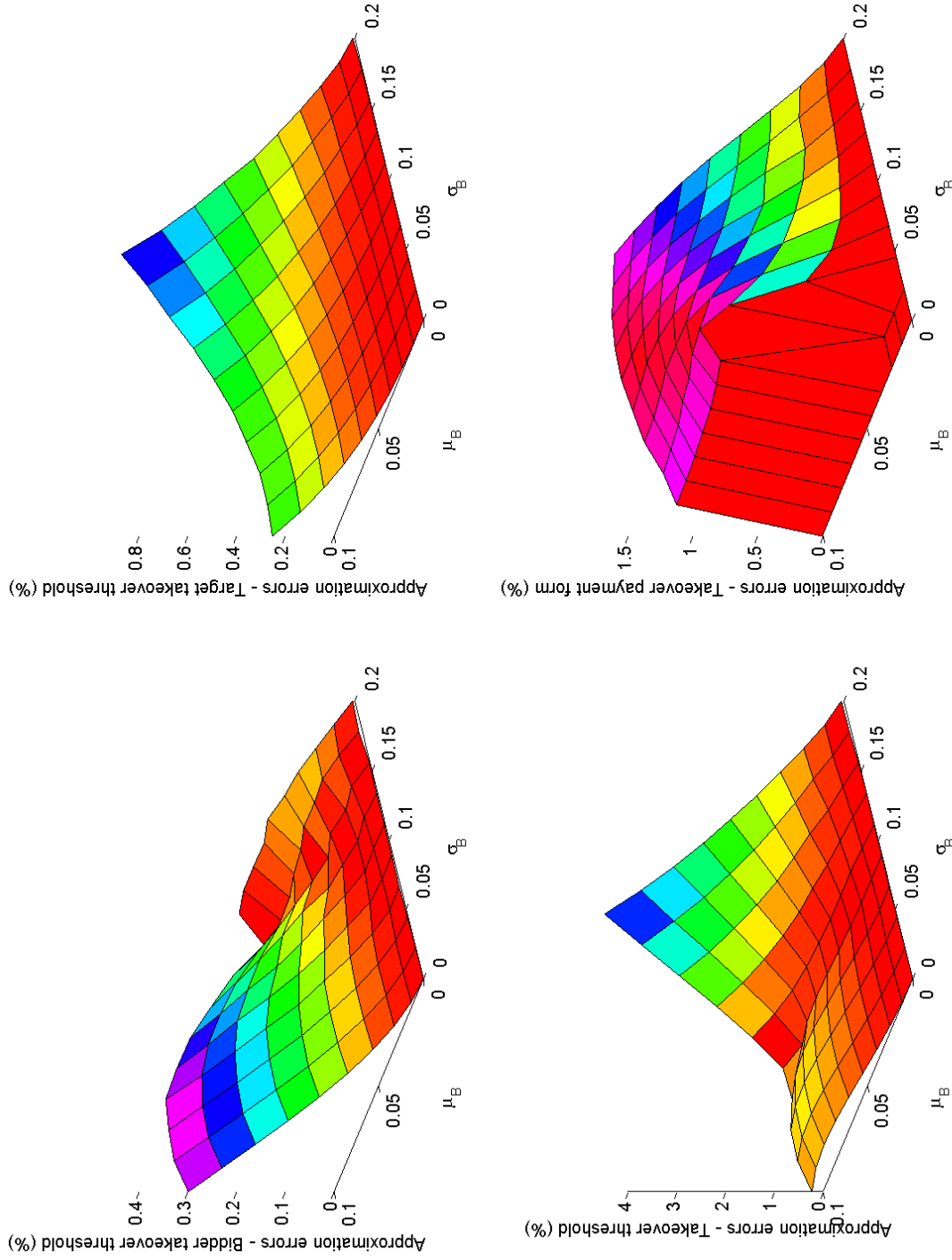
**Fig. A.9:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$



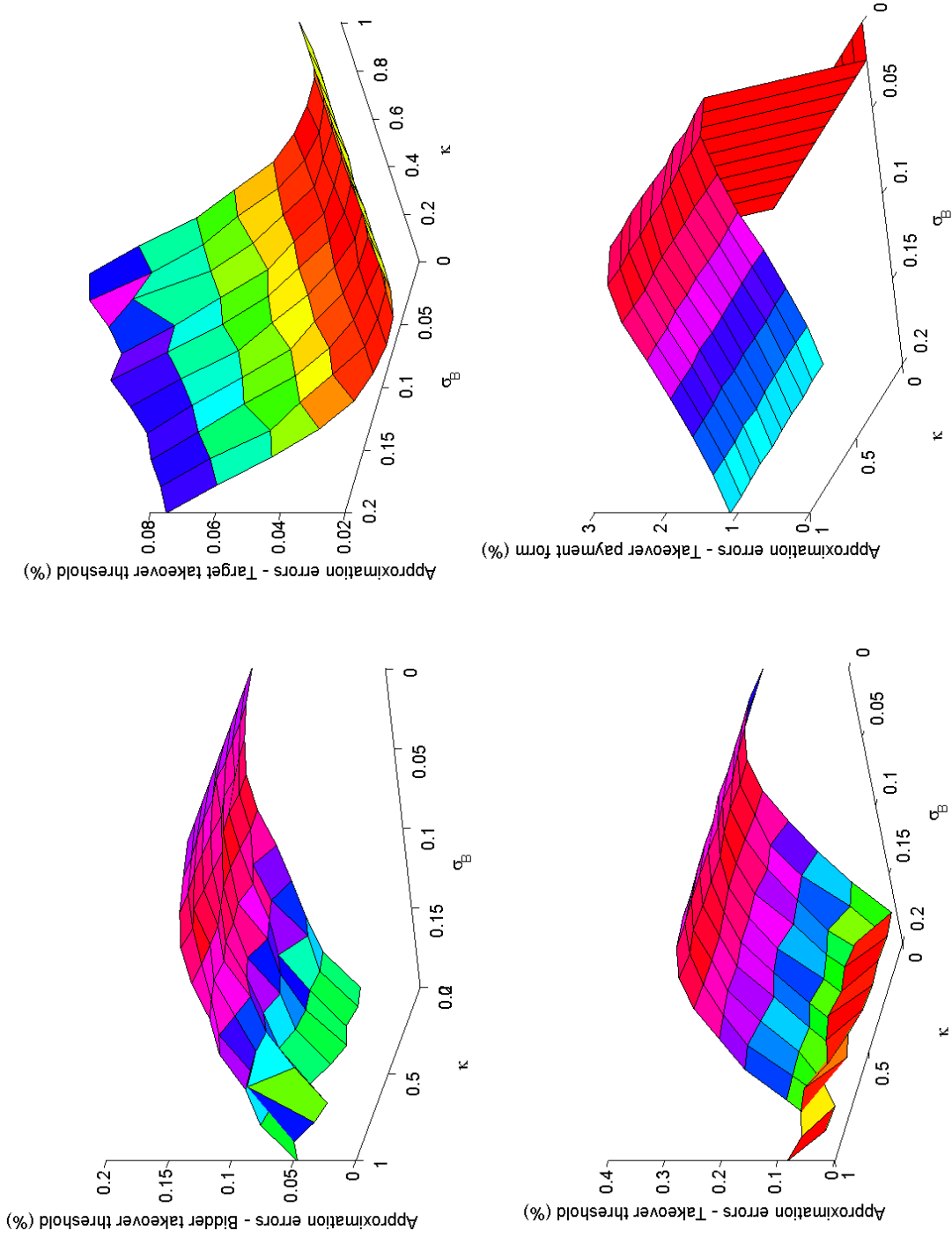
**Fig. A.10:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$



**Fig. A.11:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$

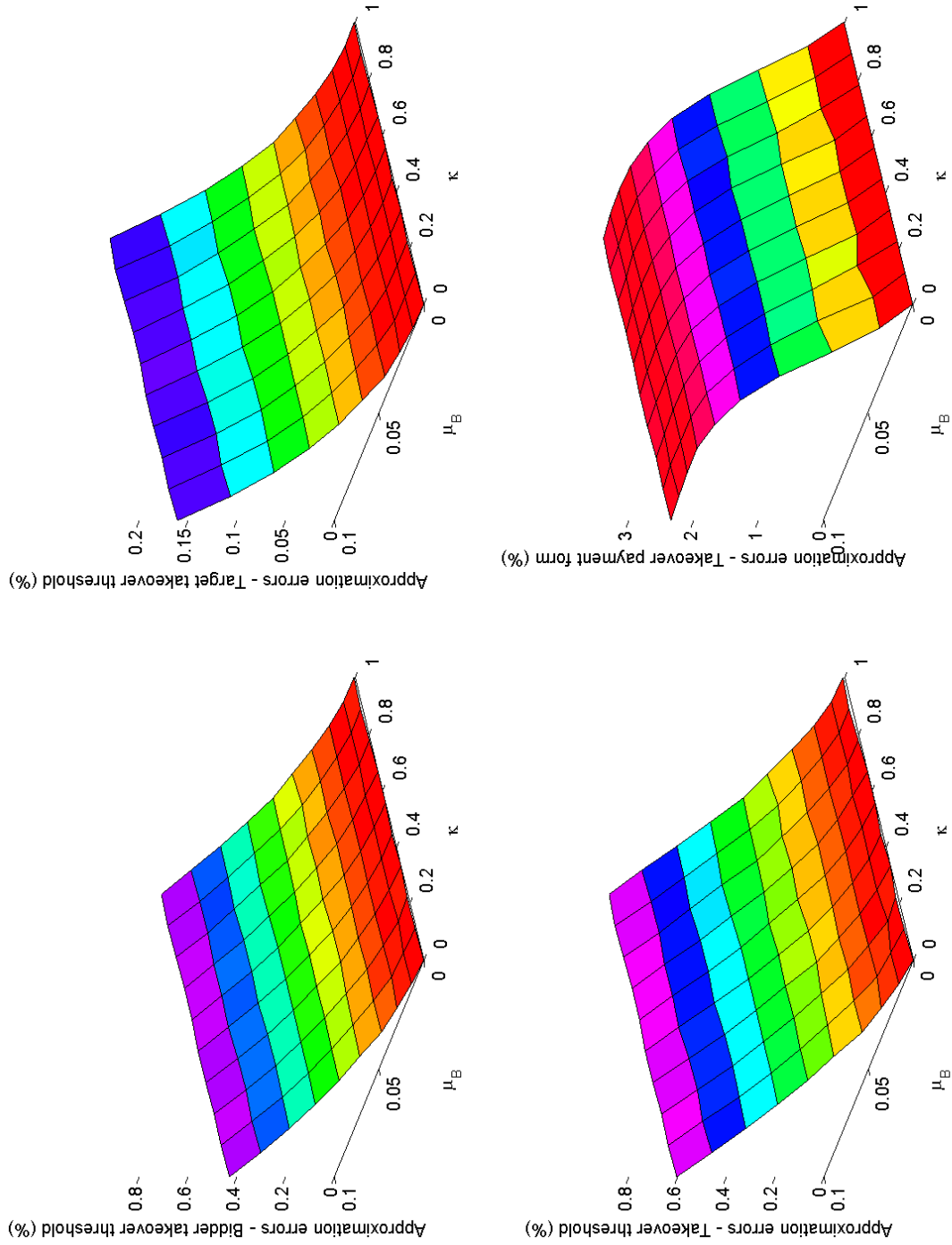


**Fig. A.12:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$

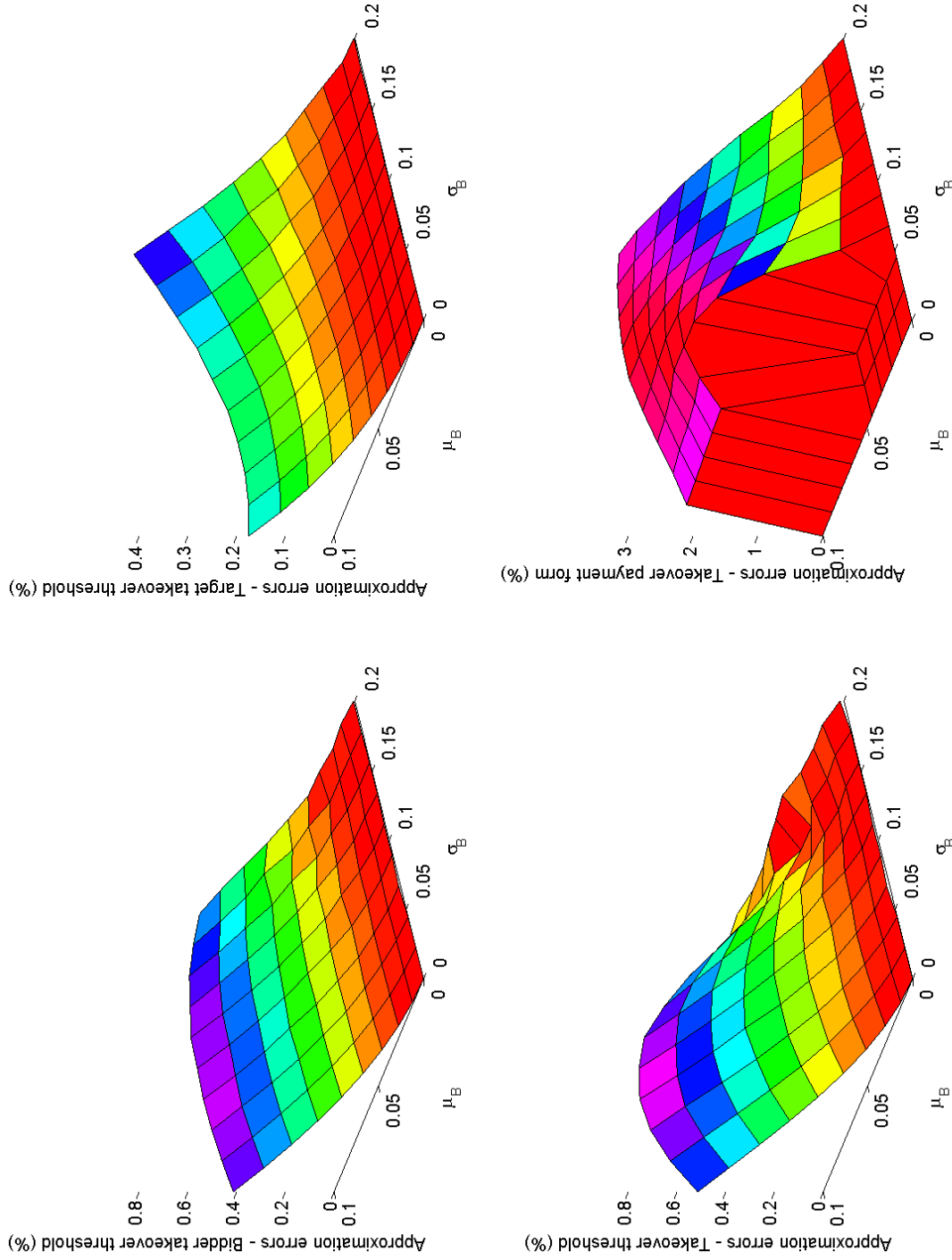


**Fig. A.13:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.06$ ,  $\sigma_T = 0.12$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$

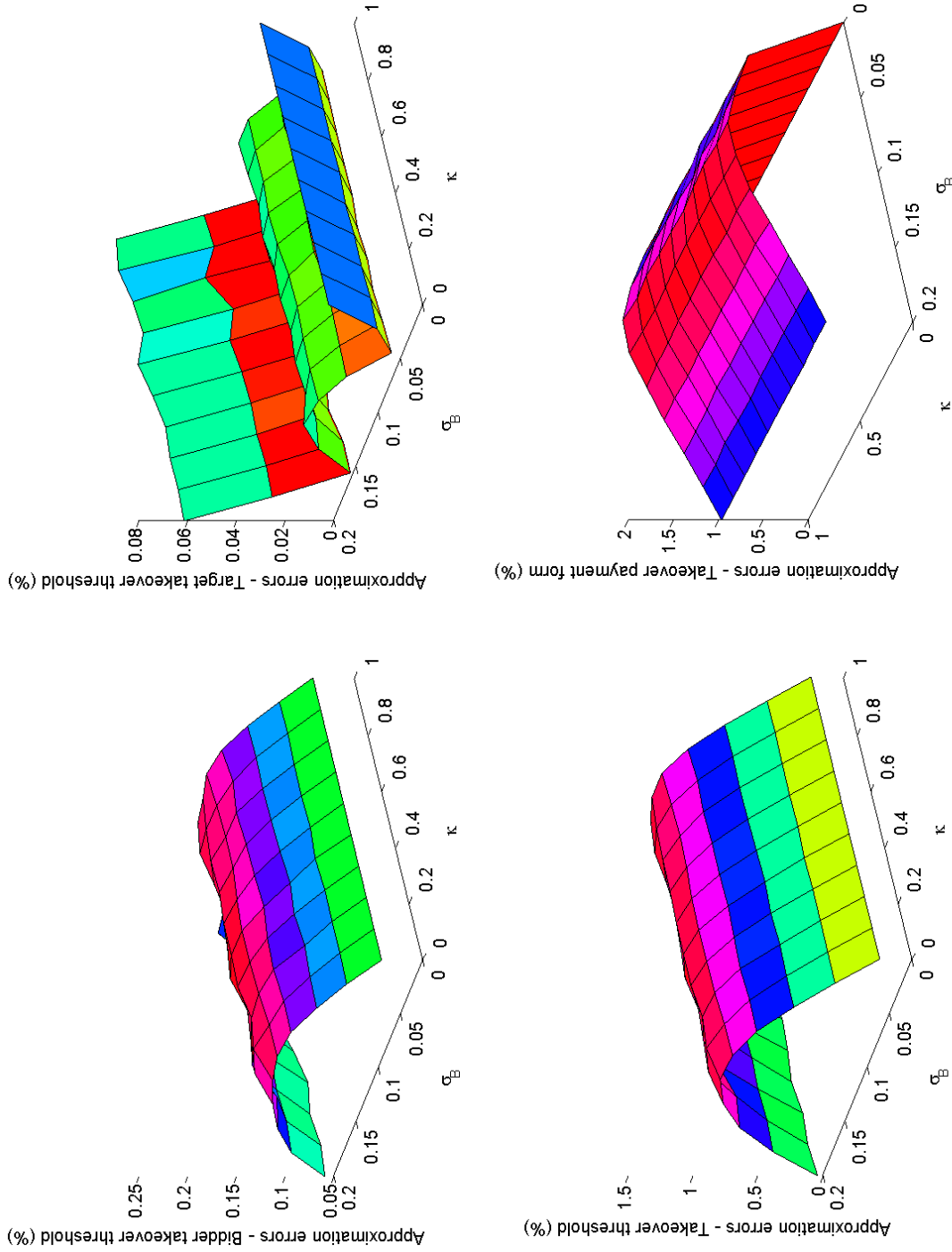




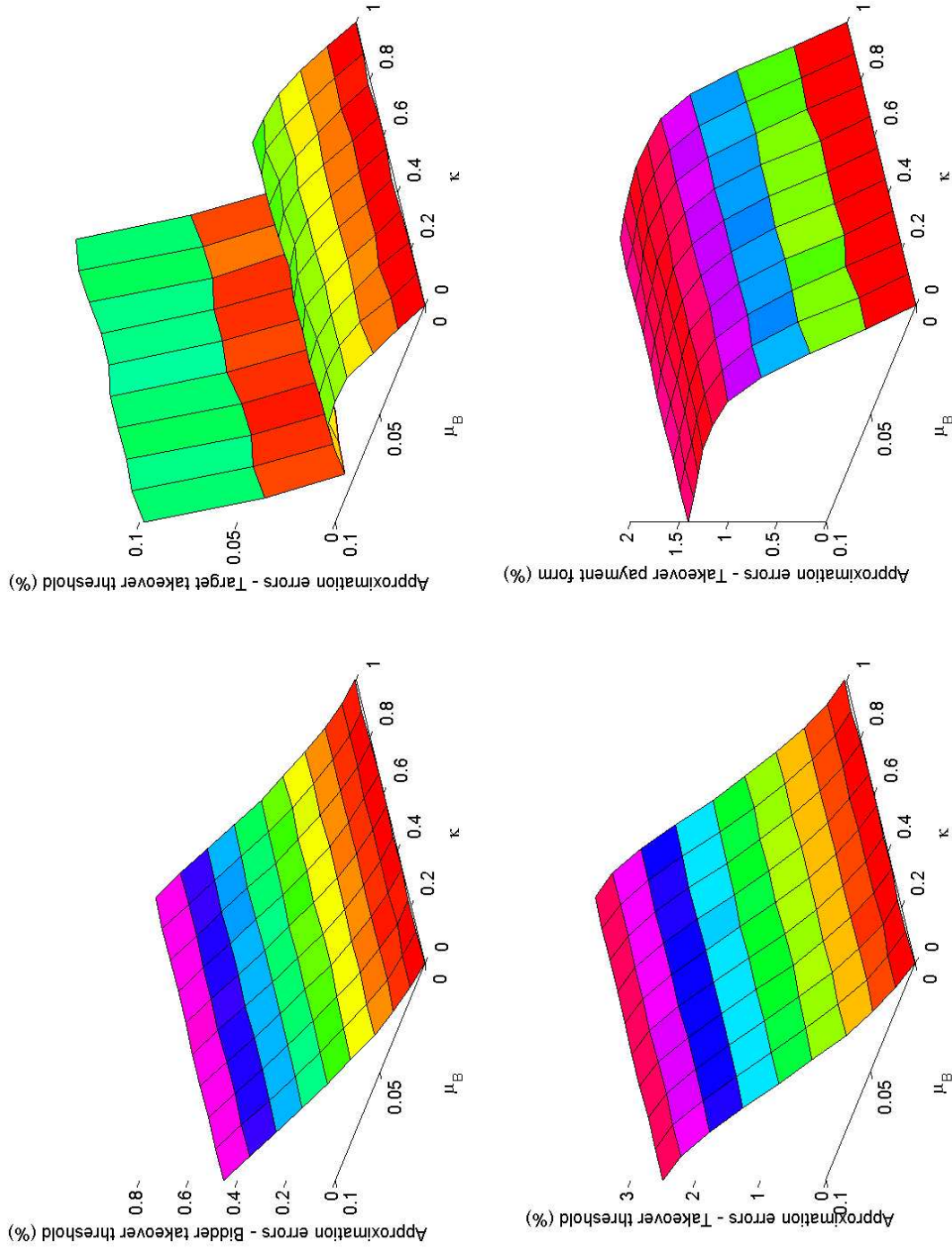
**Fig. A.14:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.06$ ,  $\sigma_T = 0.12$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$



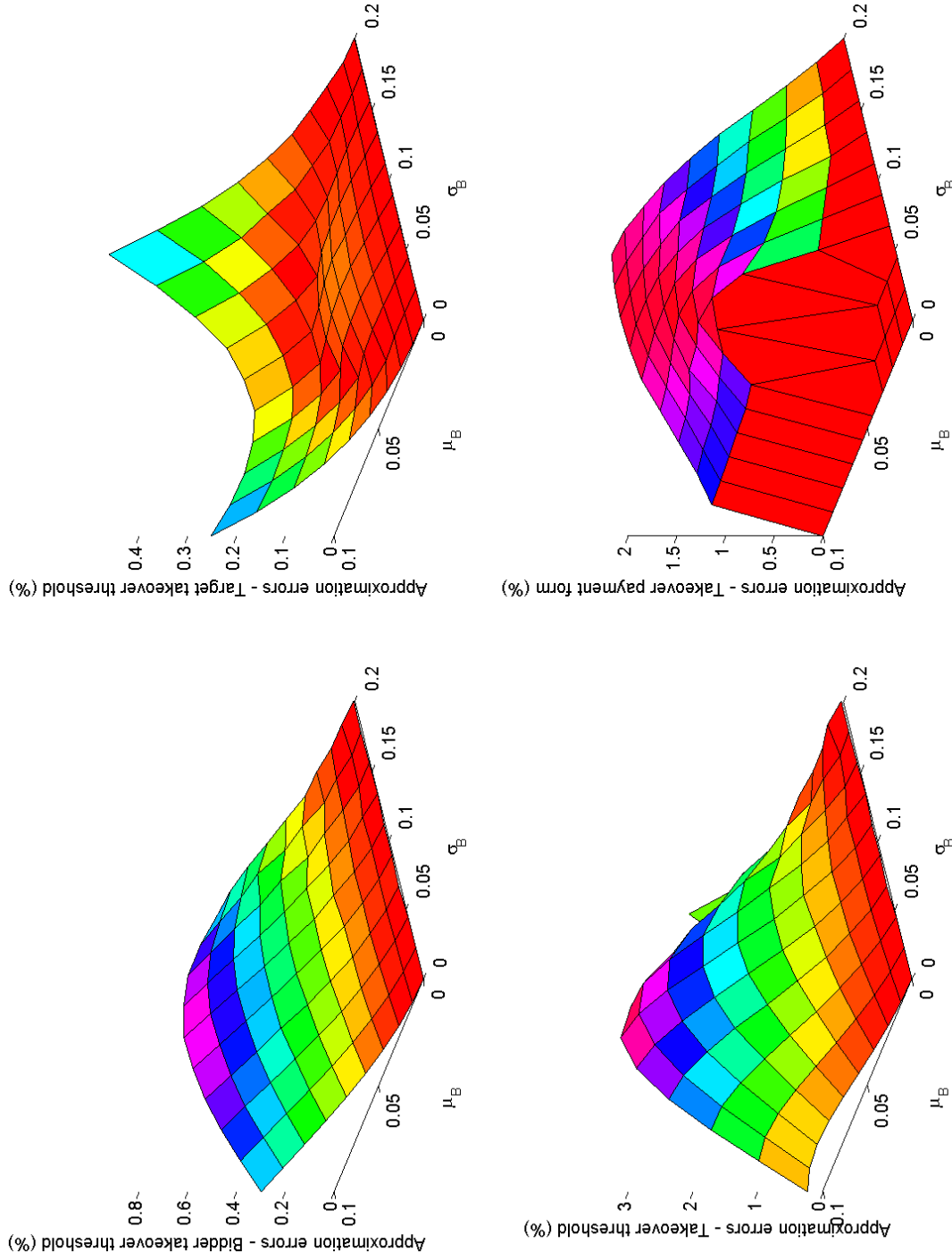
**Fig. A.15:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.06$ ,  $\sigma_T = 0.12$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$



**Fig. A.16:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.75$ ,  $r = 0.2$



**Fig. A.17:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.75$ ,  $r = 0.2$



**Fig. A.18:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.75$ ,  $r = 0.2$

solution is given by equation (3.28). As the payment form threshold can be 0% in shares, i.e., the threshold of  $\kappa$  can be 0, the errors are calculated in absolute terms, i.e., if the calculated  $\kappa$  threshold shows that 10% shares should be used in the payment form and our error show a level of 1%, the correct payment form threshold that account for the approximation errors should be either 9% or 11%. As target firm's characteristics can also influence the takeover thresholds and the payment form choice, apart from checking the impact of different  $\mu_B$ ,  $\sigma_B$ , and  $\kappa$ , target firms with different growth prospects and correlation with bidder firm are also considered. The base case for Figure A.10 - 12 includes the assumption that  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  $\kappa = 0.25$ ,  $\rho = 0.25$ ,  $r = 0.2$ . Figure A.13 - 15 considers a fast-growing and high-volatility target firm where  $\mu_T = 0.06$  and  $\sigma_T = 0.12$ . Figure A.16 - 18 explores a high correlation ( $\rho = 0.75$ ) between two takeover participants.

Again, the results are summarized as below

- The approximation errors for the bidder takeover thresholds are within the range of  $[0, 0.5\%]$  of the correct value,
- The approximation errors for the target takeover thresholds are within the range of  $[0, 0.8\%]$  of the correct value,
- The approximation errors for the overall takeover thresholds are within the range of  $[0, 4\%]$  of the correct value,
- The approximation errors for the payment form threshold are within the range of  $[0, 3\%]$  of the deal.

## A.15 Approximation errors in Chapter 4

Equations (4.4) - (4.15) specify the approximation method utilized in Chapter 4. It follows the method used in Chapter 3 where the cash payment results in a “diluted” effect to the stochastic process of newly merged firm (i.e., equations (4.14) and (4.15)). The approximation errors for this method has been discussed in the last section. Equations (4.4) and (4.5), following Leland and Skarabot (2003), introduce a new approximation approach, which essentially argues that the newly merged firm approximately follow a GBM of which the drift and volatility terms are functions of those of the bidder and target. This section focuses on checking the approximation errors for this method and concludes with the impact of the total approximation errors on the takeover thresholds for the bidder and target.

Equations (4.7) -(4.9) also specify the difference between our method and what is used in Leland and Skarabot (2003). Basically a “leverage” effect has been introduced, where the cash paid out to the shareholders of the target firm will impact the growth prospective and volatility of the newly jointed firm. In the following, it is my aim to check (1) whether it is reasonable to argue that the newly merged firm  $S^N$  (equation 4.2) follows approximately a GBM and (2) whether equations (4.8) and (4.9) provide a reasonable approximation to the drift and volatility term.

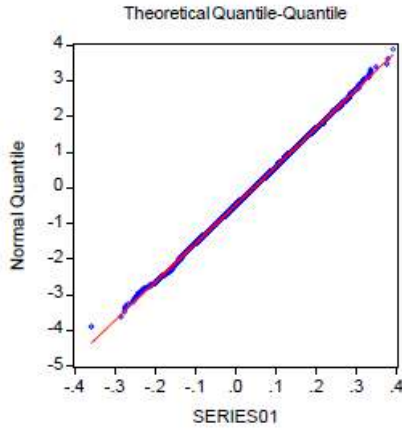
Figures A.19 and A.20 show the Quantile-quantile graph with different parameter combinations, where the log return of each simulated path of  $S_N$  has been taken and 5,000 realizations of log returns are plotted against the normal distribution. It is easily observed that in almost all cases, the log returns of  $S_N$  can be closely approximated by a normal distribution.

Again, the next step goes on to simulate the log returns of  $S_N$  as given by equations (4.8) and 4.9 (i.e., the GBM with defined drift and diffusion terms) and the exact process of  $S_N$  (i.e., a combination of two GBMs with some constant valued deducted) and compare the first to fourth moment of the returns distribution in different parameter constellations. To be consistent with the previous section, approximation errors for the first and second moments are shown in relative terms while the errors for third and fourth moments are shown as the values of skewness and excess kurtosis for the simulated exact process.

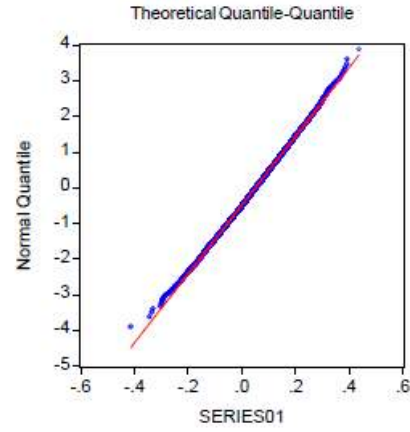
Figures A.21- 26 present the results. In general, the approximation errors for each moment depend on the parameter combination being chosen. For all the parameter constellations used in the figures,

- The approximation errors for the mean are within the range of  $[0, 2\%]$  of the exact value,
- The approximation errors for the standard deviation are within the range of  $[0, 2\%]$  of the exact value,
- The skewness of the exact process is within the range of  $[-0.1, 0.1]$ ,
- The excess kurtosis of the exact process is within the range of  $[0, 0.2]$ .

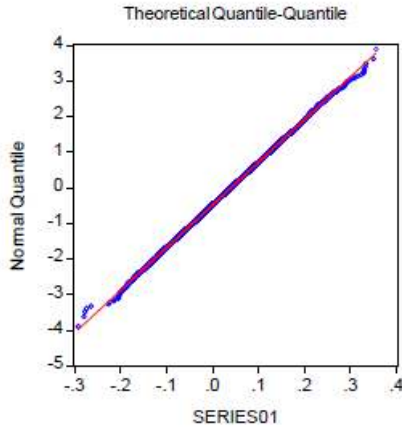
It is worth mentioning that the approximation errors for each moment become increasingly large when  $X$ , i.e., the cash paid out from the merged firm, becomes large. However even for the upper range of the  $X$  ( $X=0.5$  essentially means the half of the value of the merged firm is paid out to the target as a cash payment. It is a conservative assumption considering in most hostile takeovers, the bidder



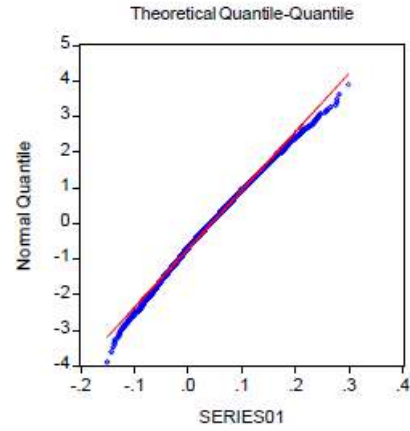
(a)  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  
 $\sigma_T = 0.06$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$



(b)  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  
 $\sigma_T = 0.06$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.25$



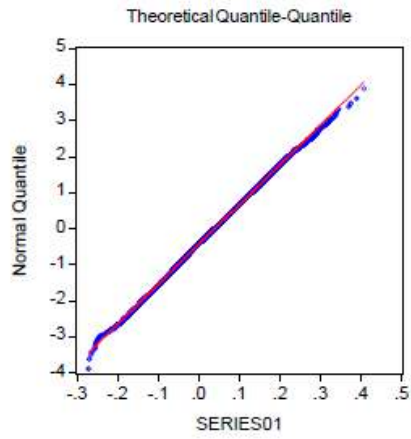
(c)  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  
 $\sigma_T = 0.06$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0$



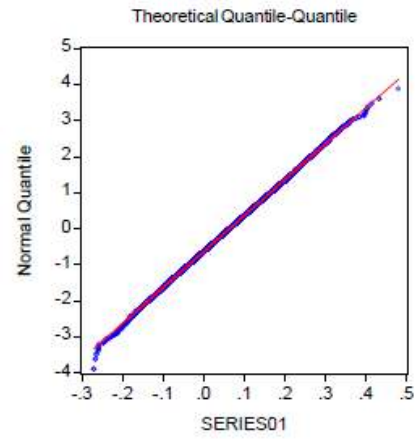
(d)  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  
 $\sigma_T = 0.06$ ,  $\rho = -0.99$ ,  $\Delta = 0.25$ ,  $X = 0.1$

**Fig. A.19:** EvIEWS Quantile-Quantile graphs - Plot against quantiles of Normal distribution

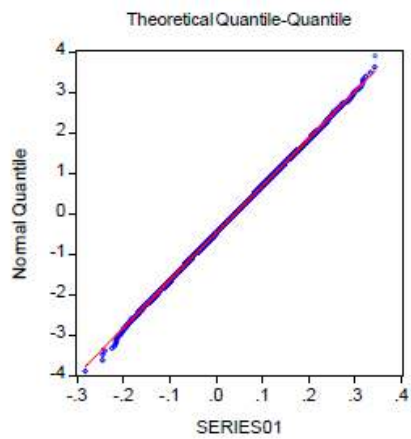




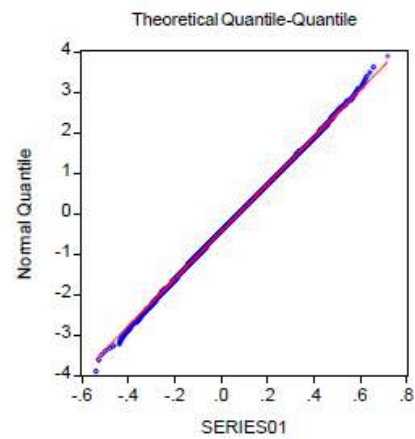
(a)  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.03$ ,  
 $\sigma_T = 0.06$ ,  $\rho = 0.99$ ,  $\Delta = 0.25$ ,  $X = 0.1$



(b)  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  
 $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$

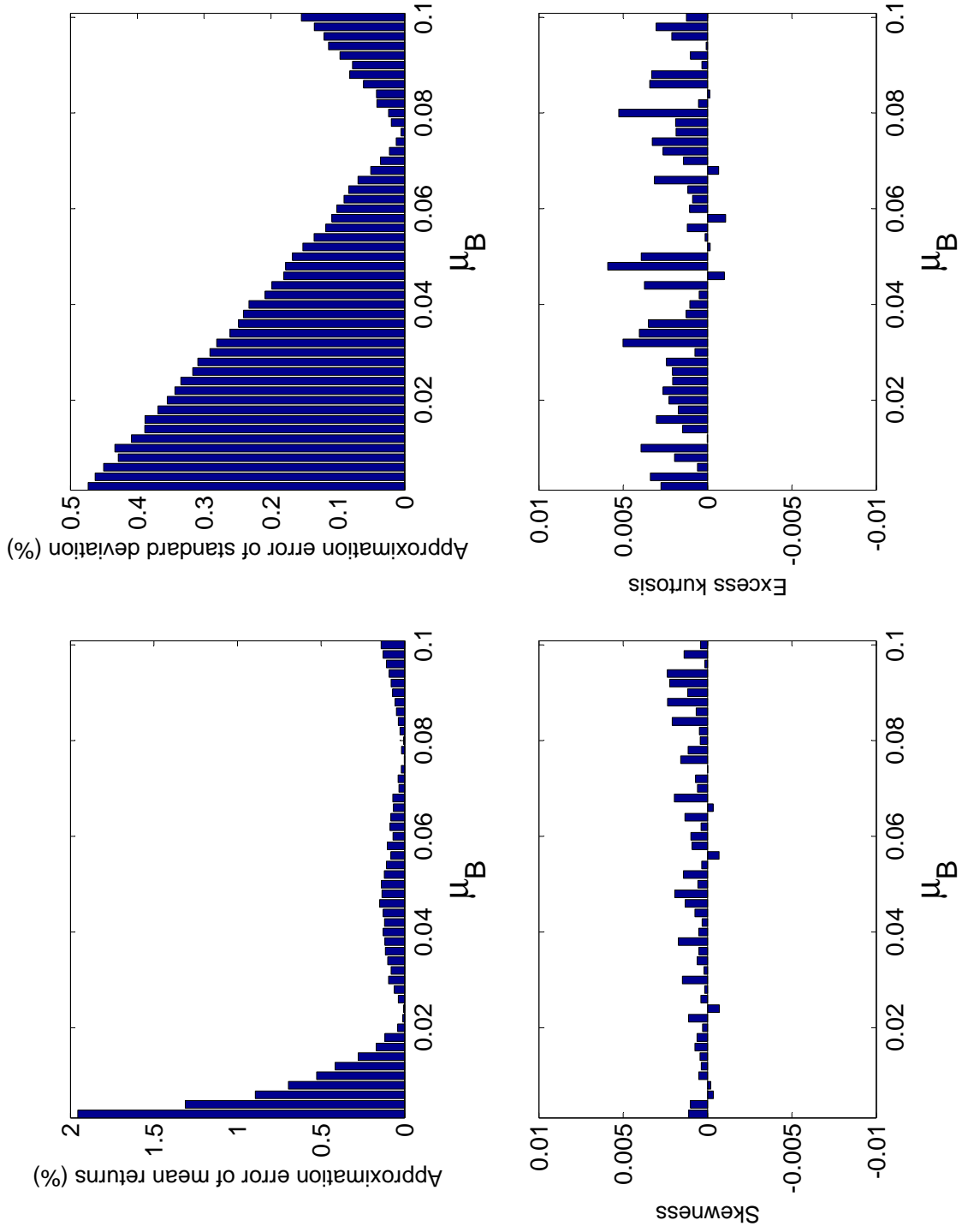


(c)  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.01$ ,  
 $\sigma_T = 0.02$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$

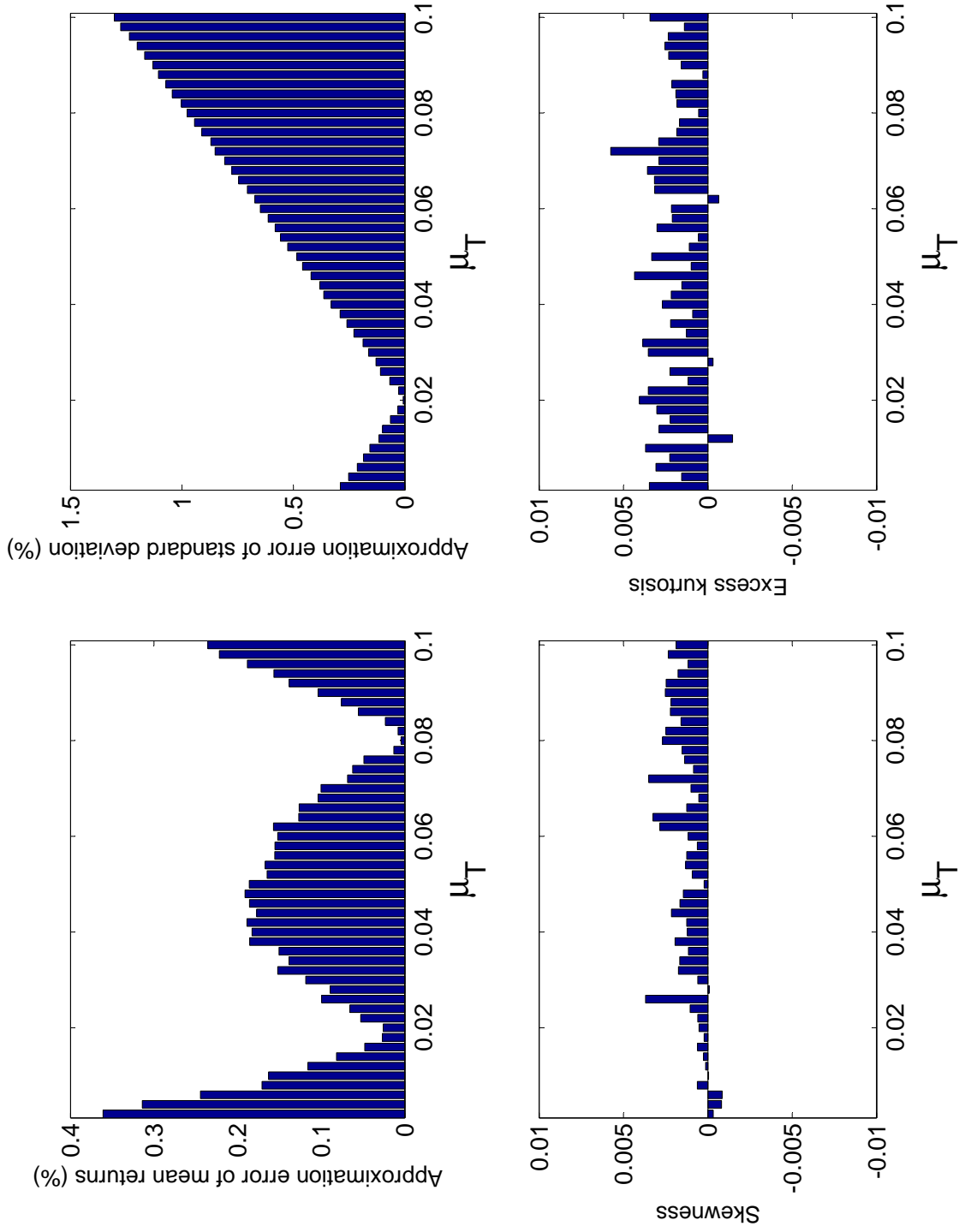


(d)  $\mu_B = 0.1$ ,  $\sigma_B = 0.2$ ,  $\mu_T = 0.03$ ,  $\sigma_T = 0.06$ ,  
 $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$

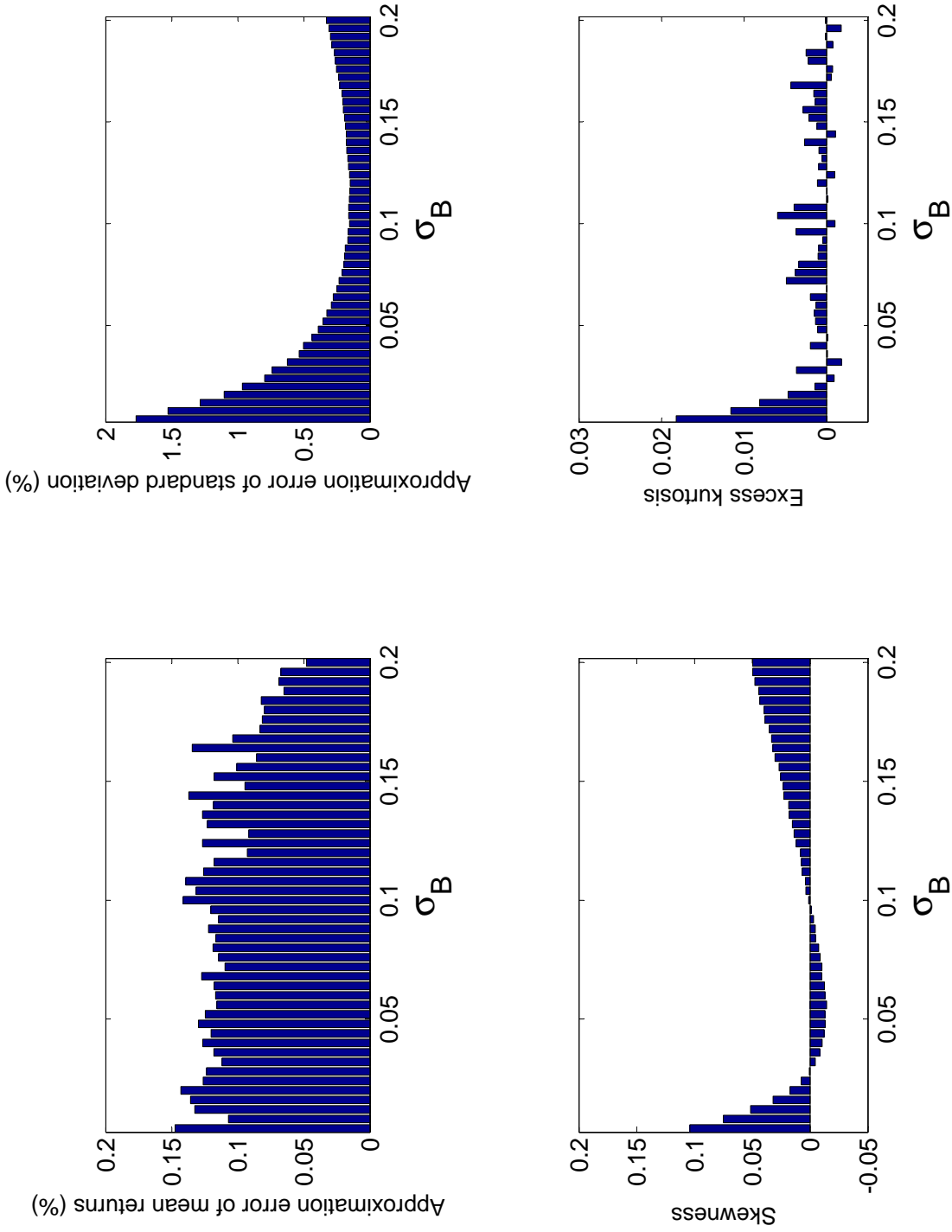
**Fig. A.20:** EvIEWS Quantile-Quantile graphs - Plot against quantiles of Normal distribution



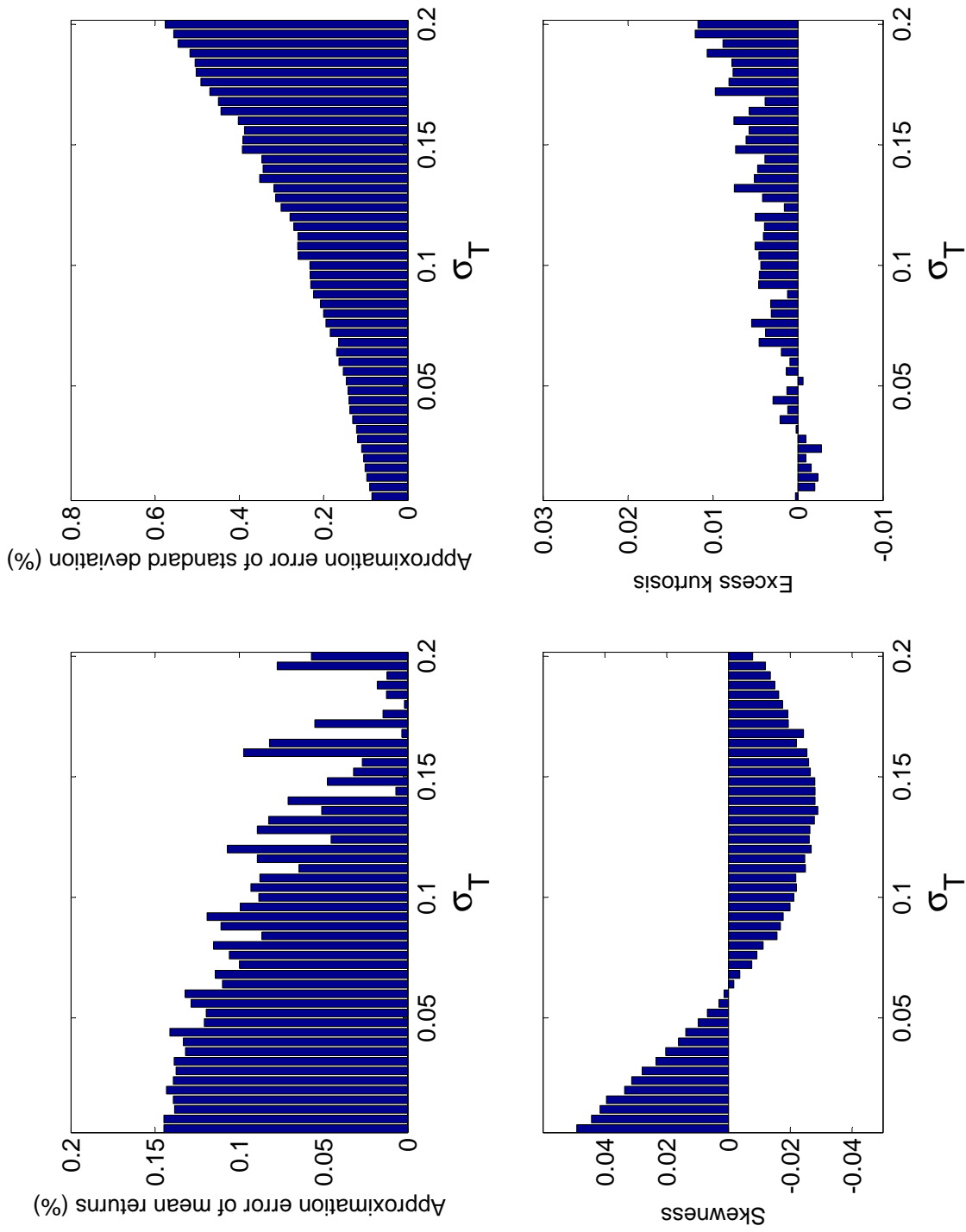
**Fig. A.21:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$



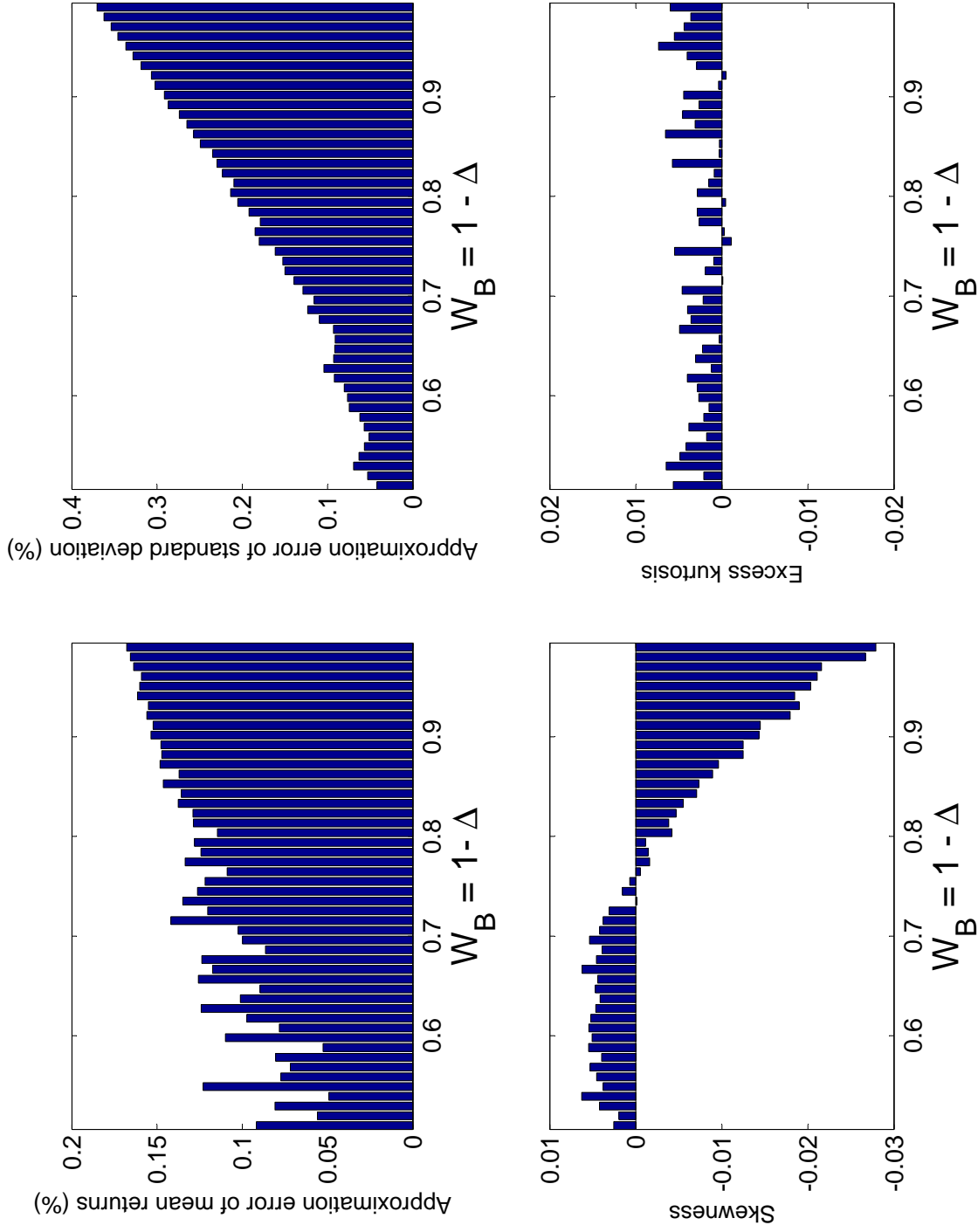
**Fig. A.22:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$



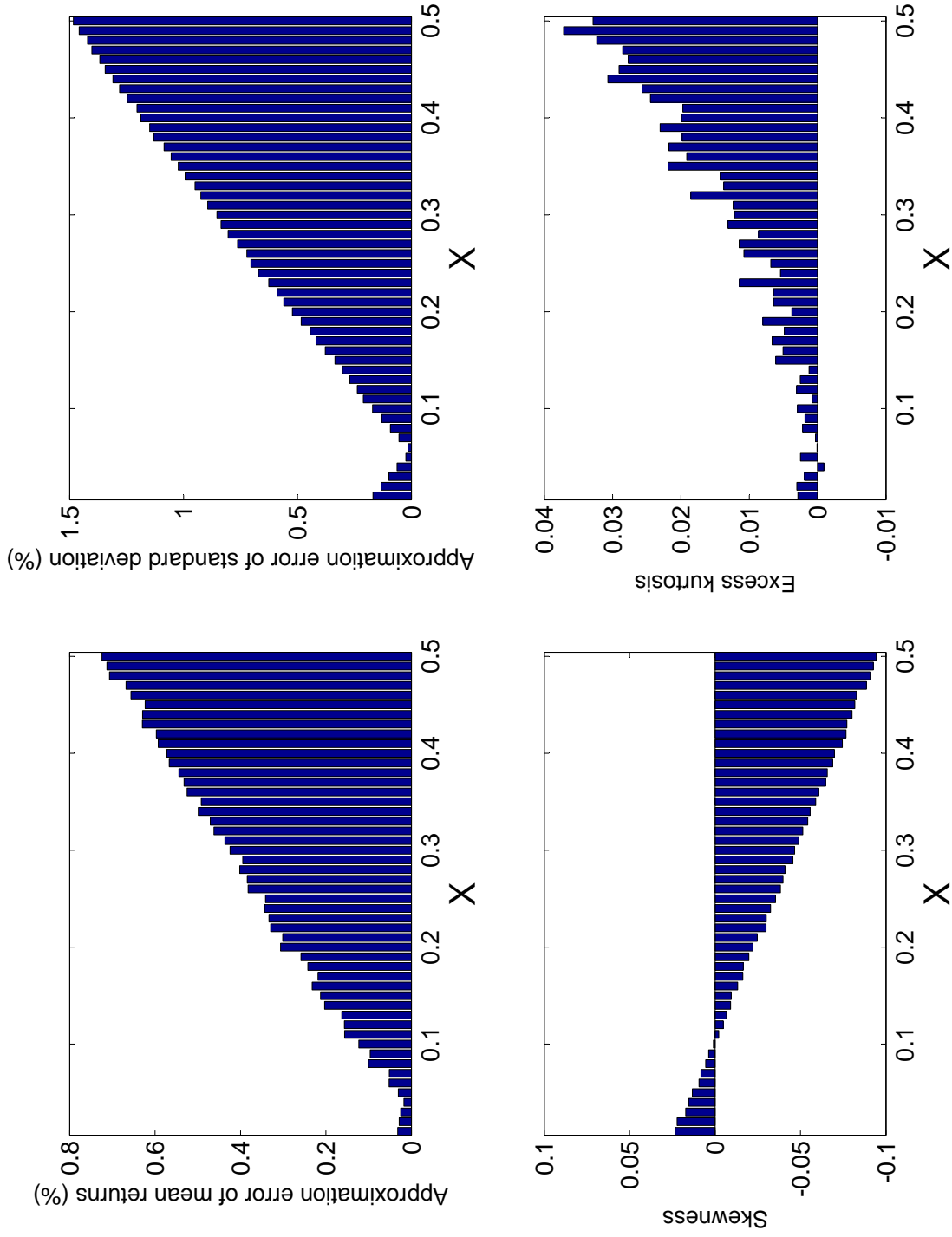
**Fig. A.23:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$



**Fig. A.24:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$



**Fig. A.25:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$



**Fig. A.26:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$ ,  $X = 0.1$

firm is much larger than the target firm, therefore even in a pure cash deal, the cash payment is unlikely to exceed more than half of the combined value of the bidder and target), the approximation errors are still modest.

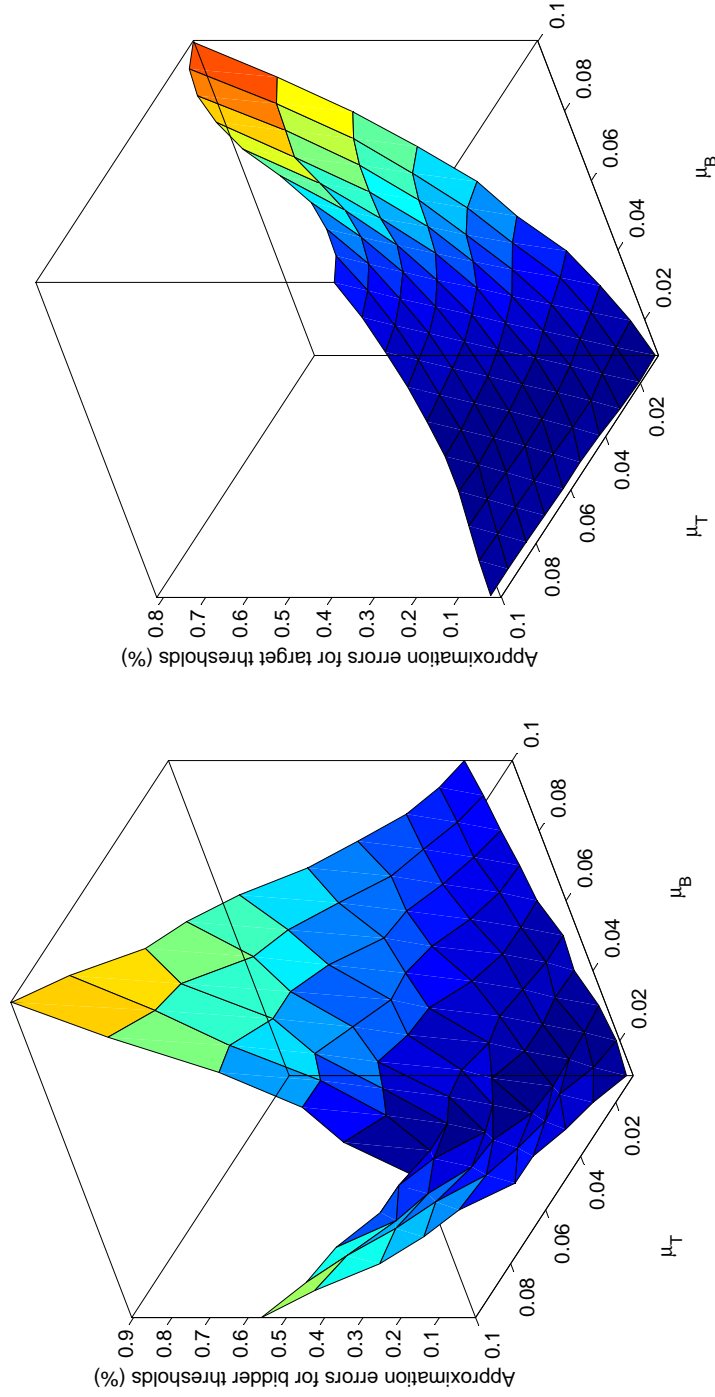
Then in the final step I intend to check the impact of the approximation errors on the takeover thresholds for bidder and target, which are the results of the real options model in Chapter 4. Again, bearing in mind that from the results shown above, the impact of non-normality should be limited and therefore I focus on checking the impact of the “biased” drift and diffusion terms.

As shown in equations (4.18) and (4.19), the takeover thresholds for bidder and target depend on the synergy effect threshold  $\alpha$  and in the mean time,  $\alpha$ , as a threshold for the takeover deal, is dependent on the threshold for each participant. The numerical methods for non-linear equations from MATLAB help solve the values for  $\alpha$  and  $\delta$ , while it is difficult to incorporate the approximation errors discussed above into the solving process. It is therefore important to clearly define the approximations errors that will be presented below.

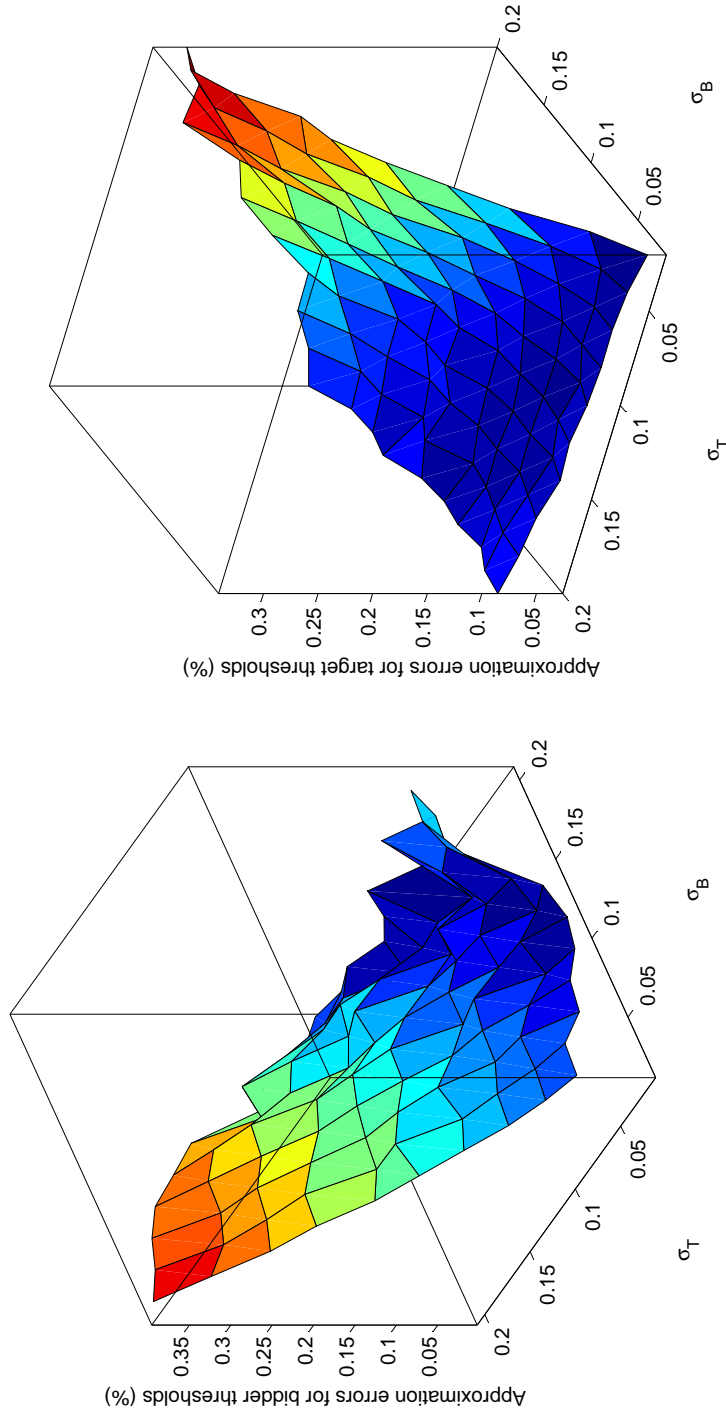
Essentially for every set of assumption (for  $\mu_B, \mu_T, \sigma_B, \sigma_T, \rho, \kappa, r, \Delta$ ), the solution for  $\alpha$  and  $\delta$ , along with the takeover threshold for bidder (i.e.,  $\frac{\beta_B}{\beta_B-1}$ ) and target (i.e.,  $\frac{\beta_T}{\beta_T-1}$ ) can be provided by equations (4.18) and (4.19). The values of  $\alpha$  and  $\delta$  and the assumption for other parameters will then be available to simulate the correct process of  $S_B^N$  and  $S_T^N$ . The simulation in the next step yields new values for equation (4.12) to (4.17) and these information will be used to calculate new takeover thresholds for bidder and target. The approximation errors presented below in figures A.27 - 29 are defined as the relative difference between takeover thresholds without incorporating the approximation errors and the takeover thresholds from the simulation. However please note that this approximation errors are simply the approximation errors for the first step of iteration because the new takeover thresholds will impact the synergy threshold  $\alpha$ , which in turn will impact the stochastic properties of involved assets. The correct takeover thresholds that fully incorporate the approximated errors (for mean and volatility) will be values that converge eventually from multiple iterations. Bearing in mind that simulation errors, which are inevitable, will make the convergence very difficult and time-consuming, I therefore present the approximation errors as a form of the error terms for the first step of iteration. It should provide a good proxy for the true approximation errors.

It is clear from the results that the approximation errors for bidder and target's takeover thresholds are less than 1% in all the parameter combinations considered. Due to the non-linear features of equations (4.18) and (4.19), it is very difficult, if not impossible, to check the impact of the approximation errors on the payment

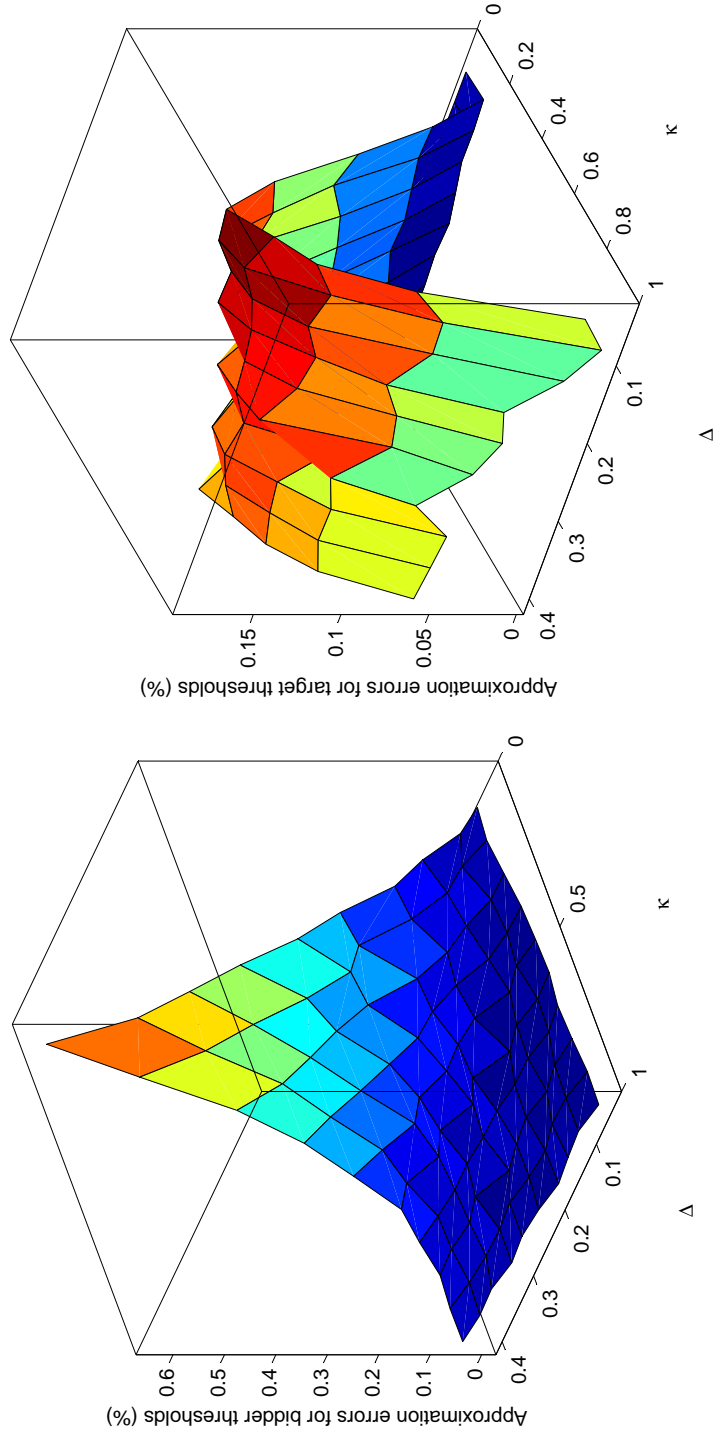




**Fig. A.27:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  $\kappa = 0.25$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$



**Fig. A.28:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  $\kappa = 0.25$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$



**Fig. A.29:** Base case  $\mu_B = 0.05$ ,  $\sigma_B = 0.1$ ,  $\mu_T = 0.1$ ,  $\sigma_T = 0.2$ ,  $\kappa = 0.25$ ,  $\rho = 0.5$ ,  $\Delta = 0.25$

form choices presented in the later part of the Chapter 4. However, a similar level of approximation errors in Chapter 3 for bidder and target's takeover thresholds result in a modest level of errors for payment form choices, it is reasonable to assume the errors in this chapter is also likely to be small.

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